THE PROBLEM OF THE SOLAR RED SHIFTS

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"THE PROBLEM OF THE SOLAR RED SHIFTS"

being a thesis presented to the University of St. Andrews in application for the Degree of Doctor of Philosophy (Ph.D.)

by Eric Gray Forbes, B.Sc.

CERTIFICATE

I hereby certify that

Eric Gray Forbes, B.Sc. has spent nine terms at research work under my supervision; that he has fulfilled the conditions of Ordinance No. 16 (St. Andrews); and that he is qualified to submit the accompanying thesis in application for the degree of Doctor of Philosophy.

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(Emeritus Professor)

DECLARATION

I hereby declare that this thesis is a record of my own studies; that it is my own composition; and that it has not previously been presented for a Higher Degree. The research described herein has been carried out initially in the University Observatory, St. Andrews; thereafter at the Osservatorio Astrofisico di Arcetri, Italy and Göttingen, Germany, under the supervision of Professor E. Finlay-Freundlich.

University and Research Training.

I first matriculated in the University of St.Andrews in October 1951 and graduated B.Sc. (Pure Science) with 1st class honours in Astronomy in June 1954. After following a one-year (non-degree) course in Aeronautics at the Imperial College of Science and Technology, London, I returned to St. Andrews in September 1955 to begin research in Astrophysics under the guidance of Professor E. Finlay-Freundlich, financed by an Andrew Bell Scholarship from the Madras College Endowment (St. Andrews) for the period of three years from October 1955-58.

From October 1957 - September 1960 I have benefited from a Special Research Grant awarded to Professor E. Finlay-Freundlich by the Department of Scientific and Industrial Research (D.S.I.R.), London, that has enabled me to use the facilities provided at the Osservatorio Astrofisico di Arcetri, Italy and at the Universitäts-Sternwarte, Göttingen to collect several series of solar spectrograms with the intention of investigating the properties of the solar red shifts. The reduction of this photographic material was carried out on the Abbé-Zeiss comparator (No.658) at Göttingen.

PREFACE.

The problem of interpreting the small systematic displacements of solar absorption lines towards longer wavelengths relative to the corresponding laboratory wavelengths - the so-called solar red shifts - has ranked as one of the most controversial problems in solar physics ever since its discovery by Jewell in 1896. The observational and theoretical difficulties which confronted the pioneer workers in this field are reviewed in Chapter I of this thesis. The year 1920 marks the beginning of a new phase in the development of the problem, since this was when Saha introduced his Ionization Theory which formed a completely new conception of the physical conditions prevailing in the solar atmosphere. At the same time, the announcement that an eclipse experiment made in 1919 appeared to confirm Einstein's prediction regarding the value of the light deflection, encouraged the belief that the gravitational red shift was implicit in the observed values of the solar red shifts; consequently, the latter were taken as resulting from a superposition of this predicted displacement upon the Doppler effects of radial currents in the solar atmosphere.

The validity of this relativity-radial current interpretation is examined in Chapter II on the basis of observational data presently at our disposal. Our survey serves to show that, although many features of the solar red shifts can be explained by assuming that the steady state of the solar atmosphere is being maintained by a microscopic circulation associated with the solar granulation, the well-established observational fact that the absolute (Sun-arc) displacements are generally in excess of the relativity value at the edge of the disk (where the Doppler effects should vanish) is in contradiction to the conventional interpretation.

This difficulty was fully appreciated by Professor Freundlich, who held the opinion that it might arise as a result of attempting to fit the observational data into a framework to which they did not rightly belong. It was this attitude which stimulated Freundlich (1954) to propose his revolutionary hypothesis that the red shifts observed in stellar spectra were produced as a result of some unknown interaction mechanism whereby light loses energy as it travels through space. Since this view did not appear to be compatible with the existence of the gravitational red shift, it was considered to be of the utmost importance to analyse the solar red shift data - which were much more reliable than those based on stellar observations - without taking it for granted that this effect existed, and determine whether the observed values supported Freundlich's interpretation. This was the original object of the present research, begun in September 1955.

The major results of the initial two years' work carried out by the writer in collaboration with Professor Freundlich at the Department of Astronomy of St. Andrews University are contained in the three reprints from the Annales d'Astrophysique which are submitted along with this thesis: they appear to confirm the validity of Freundlich's hypothesis, at the same time revealing no indication of the predicted gravitational red shift. Nevertheless, it was recognised that these conclusions were necessarily based upon a relatively small amount of observational material, and efforts were made by the writer to collect additional data with a view to investigating in more detail the observational properties of the solar red shifts, with particular reference to their dependency upon wavelength and excitation potential. Chapters III and IV are concerned with a description of the observational programme carried out with this intent at the Osservatorio Astrofisico di Arcetri, Italy, and the Universitäts-Sternwarte, Göttingen during the three-year period between October 1957 and September 1960. Much has happened during this time, however, which has caused us to re-consider the problem of the solar red shifts in an entirely different light. The present viewpoint is expressed in Chapter V.

LIST OF CONTENTS.

Page No.

CHAPTER I: HISTORICAL INTRODUCTION.

1. The Origin of the Solar Red Shifts 1

2. The Pressure Effect and the Standard Light Source 4

3. The Solar Limb Effect 10

4. Early Interpretations of the Solar Red Shifts 14

5. The Einstein Effect and the Observed Displacements 19

6. Researches on the Cyanogen (CN) Lines 22

7. The Modern Approach to the Problem 26

CHAPTER II: THE RELATIVITY-RADIAL CURRENT HYPOTHESIS.

1. The Observational Basis for St.John's Hypothesis 32

2. Evidence for Anomalies in the Allegheny Observatory Red Shifts 43

3. The Problem of the Limb Excess 49

4. The Limb Effect and St.John's Hypothesis 57

5. Schröter's Two-Stream Model and its Application to the Solar Red Shifts 68

6. Freundlich's Hypothesis 74

LIST OF CONTENTS(contd).

Page No.

CHAPTER III: AN OBSERVATIONAL INVESTIGATION OF THE CENTRE-LIMB WAVELENGTH VARIATION OF MODERATELY INTENSE FRAUNHOFER LINES IN THE VISIBLE AND NEAR INFRARED SOLAR SPECTRUM.

A. The Arcetri Observations.

1. The Arcetri Solar Tower. 85

2. Auxiliary Instruments: (a) The Guiding Device 88

(b) The "V" 90

3. The Arcetri Spectrograph 91

4. Conditions for obtaining Maximum Dispersion and Resolving Power with the Arcetri Spectrograph 93

5. Description of the Limb Effect Observations in the Visible Wavelength Range 97

B. The Göttingen Observations.

1. The Göttingen Solar Tower and Spectrograph 107

2. The Guiding 108

3. Description of the Infrared Limb Effect Observations 111

LIST OF CONTENTS (contd).

Page No.

CHAPTER IV: THE MEASUREMENT AND REDUCTION OF THE OBSERVATIONAL DATA COLLECTED AT THE ARCETRI AND GOTTINGEN OBSERVATORIES.

1. The Method of Measurement 116

2. The Consistency and General Reliability of the Measured Line Shifts 120

3. Results of Measurements of the Limb-Centre Shifts of Selected Solar Spectral Lines contained on the Arcetri and Göttingen Spectrograms 127

4. Summary of Results 151

CHAPTER V: DISCUSSION AND ANALYSIS OF THE OBSERVATIONAL DATA.

1. The Term-Dependency of the Limb-Centre Shifts 154

2. Anomalous Limb-Centre Shifts in Multiplet Lines 161

3. Features of the Solar Limb Effect Observations near 7770A 164

4. Comparison of the Present Observations with Independent Limb Effect Data 169

5. A Physical Interpretation of the Solar Red Shifts for Moderately-Intense Lines 173

6. The Relation between the Physical and Empirical Red Shift Formulae 182

7. Analysis of the Observed Wavelength Displacements of Intense Solar Absorption Lines 186

8. Conclusion 191

INDEX TO TABLES.

Table No.		Page No.
1.	The Intensity Dependency in St.John's Observations	38
2.	Dependency of St.John's Centre-Arc Shifts and Residuals of FeI Lines upon Wavelength and Line Strength (Group Values)	40
3.	Solutions of $\delta = m\lambda$ for St.John's Centre-Arc Observations	41
4.	Dependency of the A.O.-B.S. Red Shifts of FeI Lines upon Wavelength and Line Strength	48
5.	Relationship between Disk Position and Functions of the Angle of Emergence θ	99
6.	Particulars of Plates centred near 4400A	102
7.	" " " " " 5300A	103
8.	" " " " " 6100A	104
9.	" " " " " 6300A	105
10.	Description of FeI Lines measured on the Arcetri Plates	106
11.	Particulars of Plates centred near 7770A	114
12.	Description of Lines measured on the Göttingen Plates	115
13.	Results of Measurements on the FeI line λ5324.2	121
14.	Results of Measurements on AtmosphericLines	125
15.	The Limb Effect of Selected Medium-Intense FeI Lines contained in the Visible Solar Spectrum	152
16.	The Limb Effect of Selected Medium-Intense Fraunhofer Lines contained in the Near-Infrared Solar Spectrum	153

INDEX TO TABLES (contd)

Table No.		Page No.
17.	The Dependency of the Limb-Centre Shifts on χ_L for Medium-Strong FeI Lines in the Visible Wavelength Range	157
18.	The Observed Limb Effect of Medium-Strong FeI Lines originating from Different Levels of Excitation	160
19.	The Observed Limb Effect for Lines belonging to the Multiplet Transition $a^5P - y^5D^o$ ($\bar{\chi}_L$ = 2.19 eV)	162
20.	The Observed Limb Effect near 7770A for Different Elements	168
21.	Comparison of Velocity Shifts derived from Relative and Absolute Red Shift Measurements of Moderately-Intense FeI Lines	180

CHAPTER I.

HISTORICAL INTRODUCTION.

1. The Origin of the Solar Red Shifts.

The basic assumption underlying all comparisons between solar and terrestrial wavelengths is that conditions of local thermodynamical equilibrium, or pure absorption, prevail in the outer layers of the Sun where the Fraunhofer lines are produced. The re-emission of absorbed radiation is then governed by Kirchhoff's Law, and at any given frequency the absorption and emission process are reciprocal. Under these circumstances, a Fraunhofer line should appear on a spectrogram in precisely the same position as the corresponding emission line excited under equilibrium conditions in a laboratory light-source. In practice, however, this is never found; for there exists a well-established kinematical effect which must first be taken into account before any such comparison is made. This is the Doppler effect, which exhibits itself in radial velocity observations as an increase in the observed wavelength when the relative motion between the source (in this case the Sun) and the observer is one of recession, and as a corresponding decrease when it is a velocity of approach. Consequently, in solar wavelength comparisons, allowances must be made for the Earth's orbital and axial motions; for these motions can combine to produce a line-of-sight velocity equivalent to a change in wavelength of as much as 5 milli-Angstroms (mA), although in general it will be less. A further correction for solar rotation may also be required: the wavelengths of spectral lines observed on the East and West sides of the Sun's polar diameter suffer violet and red displacements respectively, due to the relative motion of the solar gases approaching and receding from us. The magnitude of this rotational effect varies with position on the Sun's disk, being effectively zero at its centre and along the polar diameter, but amounting to roughly $\pm$ 40 mA at the ends of the equatorial diameter. In all quantitative comparisons of solar and terrestrial wavelengths it may be taken for granted that the above corrections have first been applied. Evidence for the existence of localised convection currents within the solar atmosphere - which also give rise to Doppler displacements - will be considered in detail later.

When Rowland[1)] began his extensive mapping of the Fraunhofer spectrum in 1887, the Doppler effect was thought to be the sole cause capable of producing displacements of spectral lines. He and his collaborator Jewell were therefore surprised to find, when comparing solar and electric arc wavelengths after applying the known corrections, that solar lines appeared displaced by several milli-Angstroms - generally towards the red - relative to the corresponding arc lines. Rowland, appreciating the inadequacy of his equipment for a task demanding such high precision, felt justified in attributing these small discrepancies to accidental errors, and to a possible systematic error due to unequal illumination of the grating. Moreover, the poor resolution afforded by the small solar image which he had been obliged to use caused Rowland to doubt whether the light projected on to spectrograph slit had indeed been received exactly from the centre of the disk: thus the Doppler effect of the solar rotation may not have been eliminated. Apart from these observational errors, he was also aware that small differences in the wavelength measurements could also have been produced by the presence of turbulent conditions on the solar disk.

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1) Rowland, H; Preliminary Table of Solar spectrum Wavelengths (Carnegie Institute of Washington), 1895.

Jewell, on the other hand, was not prepared to accept this opinion. He carefully repeated all the measurements and discovered that the displacements between solar and electric arc wavelengths vary in amount from element to element, from line to line belonging to the same element, and even among observations of the same spectral line taken on different photographic plates. Thus it became apparent that accidental errors alone were unable to account for the observed discrepancies. Jewell[1] was therefore the first to establish the existence of systematic shifts of a physical character in solar wavelength determinations: he thought that they could not be Doppler effects, since they were not directly proportional to wavelength and appeared to increase with line intensity. The essential feature of Jewell's results was that, with few exceptions, all solar wavelengths appeared displaced to the red end of the spectrum relative to the corresponding emission lines formed in the electric arc. Appropriately, these wavelength differences are called "red shifts", and it was to the Sun-arc displacements that this term was originally applied.

2. The Pressure Effect and the Standard Light Source.

At the same time as Jewell made his discovery (1896),

1) Jewell, L.E.: Ap.J. 3, 89, 1896

Humphreys and Mohler[1] found that shifts with characteristics similar to those observed on the Sun are produced in arc spectra when the pressure is increased beyond atmospheric; pressure, more so than temperature, appeared to be the basic factor determining the shifts. Some features of this pressure effect which these two investigators established are: with increasing pressure the spectral lines in the arc broaden asymmetrically and are displaced towards the red; also, the increase in wavelength is proportional to the increase in pressure and varies from line to line. These results were limited to pressures in the range from 1 - 14½ atmospheres.

A more significant contribution to our knowledge of the character of pressure shifts was soon forthcoming from the interferometric investigations of Fabry and Buisson[2], who immediately found from a study of FeI lines that spectral lines should be classified into three groups according to whether their Sun-arc (in air) displacements are large and to the red, small and to the red, or towards the violet. They discovered the same distinction when comparing lines in the electric arc (in air) with those in a vacuum arc. The behaviour of the individual FeI lines when the pressure was

1) Humphreys and Mohler; Ap.J. 3, 114, 1896.
2) Fabry and Buisson; C.R., 148, 688, 1909.

increased up to one atmosphere was found to be very complex, some lines broadening considerably and being shifted asymmetrically towards the red or violet; on the other hand, all appeared to be finer and of nearly the same width (about 30 mA) when produced in the vacuum arc.

In an attempt to provide a quantitative estimate of the effect of pressure under different conditions of state, A.S. King began to study the effect of pressure on different elements: viz. iron, titanium, calcium, etc., contained in the spectra produced in an electric furnace. His extensive researches[1)] have been principally concerned with the shifts of iron lines, from which he deduced that the pressure effect does not depend on: the temperature of the furnace; the density of the iron vapour in which the lines are produced; and the presence of vapours belonging to foreign gases. Two positive results of King's investigations, however, are:

(i) For a given increase in pressure, the shifts of the neutral (arc) lines produced in the furnace and in the electric arc are identical.

(ii) The observed displacements of the ionized (enhanced) lines in furnace spectra are almost twice as large

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1) King, A.S.: Ap.J. 27, 353, 1908; 34, 37, 1911; 35, 183, 1912; 38, 315, 1913; 40, 213, 1914; etc.

as those measured in the arc.

These discoveries left no doubt that the effect of pressure depends upon the nature of the light source employed. It gradually became known as the result of numerous researches by such eminent observers as Gale and Adams[1)], Goos[2)], St. John and Ware[3)], Royds[4)], St. John and Babcock[5)], Gale and Whitney[6)], and others, that absolute wavelength measurements by different observers were not equally reliable, agreement being obtained only among lines which broaden symmetrically under pressure.

The major result of all these experiments carried out with furnace and arc spectra was the realisation that many of the anomalies found in earlier comparisons of Fraunhofer lines with the lbboratory wavelengths were due to the pressure effect - whose features we have described - and to a so-called "pole-effect"; the latter could be rendered negligible by selecting for measurement only those lines which were known to be insensitive to pressure changes (i.e. those which do not broaden appreciably under an increase in pressure) and

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1) Gale and Adams; Ap.J., <u>35</u>, 10, 1912; <u>37</u>, 391, 1913.
2) Goos; Ap.J., <u>38</u>, 141, 1913.
3) St.John and Ware; Phys.Rev., <u>1</u>, 67, 1913.
4) Royds,T.; Kod.Obs.,Bull.,No.40, 1914; No.54, 1916.
5) St.John and Babcock; Ap.J., <u>42</u>, 231, 1915; <u>46</u>, 138, 1917.
6) Gale and Whitney; Ap.J., <u>43</u>, 101, 1916.

by choosing suitable conditions for the functioning of the electric arc: viz. the distance between the electrodes and the intensity of the exciting current. The physical explanation of the above results is now clear, since we know that the observed pressure and pole-effects constitute features of the Stark effect[1] which produces changes in atomic energy levels by splitting them into several components in the presence of an electric field. Under not too high a resolution, the multiplicity of the transitions among the components of both levels has the effect of broadening the spectral line asymmetrically, thus producing a shift in the "centre-of-gravity" of the line profile: the amount of this shift depends upon the strength of the electric field, which is determined by the concentration of charge at different distances from the electrodes.

The outcome of these investigations of pressure and pole-effect was the adoption in 1922 by the International Astronomical Union (I.A.U.) of the following recommendation[2]:-

> "In order to obtain lines of constant wavelength, constant intensity distribution and adapted to high orders

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1) Stark, J.: Phys.Zeit. 10, 902, 1909.
2) Trans.I.A.U., 1, 36, 1922.

of interference, the adoption is recommended of the Pfund arc[1] operated between 110 and 250 volts, with 5 amperes or less, at a length of 12-15 millimetres used over a central zone at right angles to the axis of the arc, not to exceed 1 - 1.5 millimetres in width, and with an iron rod 6 - 7 millimetres as the upper pole and a bead of oxide of iron as the lower pole."

The object of standardising the conditions is two-fold: one aim is to minimise pole-effect and - as far as possible - to eliminate it; the other is to ensure uniformity in arc measurements among different observers. Many observers have since worked with standard vacuum arcs, or arcs in air and applied pressure corrections to obtain the effective wave-lengths "in vacuo", believing that by so doing they have eliminated all effects of pressure; however, in view of the anomalies found by King between pressure shifts of ionized lines excited in furnace and arc spectra, and the complexity of the Stark effect, one must always be cautious not to place too much reliance upon values of the Sun-arc displacements, especially those which have been derived from a source that does not comply with the I.A.U. specifications. This

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1) Ap.J., 27, 297, 1908 (fig.1).

realisation is extremely important when we come to assess the validity of attempts to account for the observed solar red shifts.

3. <u>The Solar Limb Effect</u>.

The difficulties which we have been discussing do not apply to wavelength comparisons made between spectra from different regions of the Sun's disk, since by making such relative measurements all reference to terrestrial wavelength standards is avoided. The first attempt at a quantitative comparison between wavelengths of the spectrum of the Sun's limb and those at its centre were made by Halm[1)] in 1907, although it had been known long before that time that the two spectra present quite a different appearance. Previous to this investigation, Halm[2)] had made accurate spectroscopic observations on the two FeI lines at 6302 A for the purpose of establishing the Doppler effect of solar rotation, and with this knowledge at his disposal, he was able to calculate the shifts of the same lines relative to the two atmospheric oxygen lines in their immediate vicinity, at selected disk positions on different solar radii. As a result of observing the centre and points at distances of $\frac{1}{2}R$, $\frac{3}{4}R$, R from it - R

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1) Halm, J.: Astr.Nachr., <u>173</u>, 273, 1907.
2) Halm, J.: Trans-Royal Soc.Edin., <u>41</u>, (1), 1904.

being the solar radius - he discovered that the wavelengths at the limb exceeded those at the centre by about 12 mA, while the intermediate measures indicated that the increase towards the limb was gradual. Through this work Halm definitely established the existence of a hitherto unknown shift of a physical nature in the wavelengths of Fraunhofer lines, to which he gave the name "limb-effect".*) Fabry and Buisson, applying their interferometer to the limb-centre shifts of 14 spectral lines around 4400A, confirmed that Halm's limb-effect was not peculiar to the two FeI lines and the wavelength region to which his investigation was restricted; but no definite conclusions could be drawn concerning the characteristics of this newly-discovered phenomenon until more observational material had been accumulated with much better resolution and improved equipment. These requirements were soon fulfilled, however, when the 60-foot tower telescope was installed at the Mount Wilson Observatory.

Walter Adams, using this instrument in conjunction with a high-dispersion grating spectrograph, observed the limb-centre shifts of 470 Fraunhofer lines belonging to various (neutral and ionized) elements in the spectral range between

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*) In this thesis, we shall always use this term in Halm's original sense: viz. to denote the observed wavelength variation from centre to limb.

3741A and 6573A [1]. The effect of solar rotation was eliminated by averaging the observed wavelengths at the East and West limbs in 0° heliographic latitude, a suitable arrangement of small prisms enabling the two spectra to be photographed simultaneously on either side of the spectrum of the central portion of the 170 mm diameter solar image. Due care was naturally taken to avoid other possible sources of systematic error. Two fundamental features which Adams discovered from an analysis of his extensive data were:-

(i) The limb-centre shifts are directly proportional to wavelength;

(ii) The mean relative displacement for a group of ionized lines averages higher than that for an equal number of neutral lines belonging to the same element.

The first of these is in accordance with the interpretation of the relative wavelength differences as velocity shifts, and so Adams considered the possibility that the second characteristic might be a Doppler effect between the hotter and cooler parts of the photosphere observed in the solar granulation; however, since the measures of the FeI lines - which constituted the bulk of his material and on which the weight of his conclusions had necessarily to be based - showed a tendency to favour a higher-power wavelength dependency, he

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1) Adams, W.S.; Ap.J., <u>31</u>, 30, 1910.

felt that pressure might be the dominant cause producing the shifts.

Now although Adams' programme yields useful information concerning the variations with wavelength and element, it affords no knowledge of the functional form of the wavelength variation across the disk. The importance of such measures for determining the contribution of the effect of radial currents to the observed shift was fully realised by Evershed and Royds[1], who investigated this problem by projecting a primary solar image of 14 mm diameter (enlarged to 28 mm) upon the slit of a high-resolution grating spectrograph - centering it precisely so that the wide spectrum obtained represented a diameter of the Sun - and measuring the relative displacements of solar wavelengths along a radius at 2mm intervals as far as 10 mm from the centre, and thereafter into 1 mm intervals, to 13 mm; the final measures referred to a point 0.25 mm inside the limb. The overall results of this research are that weak (low-level) lines of FeII and TiII exhibited a limb-effect which could be well-represented by a cosine curve; three lines in the red of mean Rowland intensity 5 showed a considerable departure from this simple geometrical relation; and two strong (high-level) FeI lines of intensity 8

1) Evershed, J. and Royds, T; Kod.Obs.,Bull.No.49, 1916.

showed a much greater departure, in the sense that the increase to the limb is less pronounced for the strong lines than for the weak ones.

4. Early Interpretations of the Solar Red Shifts.

Let us now examine the interpretations put forward by the various observers to explain the results of their observations of the solar red shifts. When Jewell first made his discovery, he thought that the cause of the displacements might be found in the different temperature and density of matter prevailing in the Sun, compared with the conditions employed to excite spectral lines in the laboratory. Being unable to find evidence for any such changes among the laboratory spectra, he suggested that the high pressure then thought to exist in the solar atmosphere might be responsible for producing the anomalous shifts, a belief which seemed to be justified in view of the similarity in the characteristics of the pressure shifts measured in the electric arc. Together with Mohler and Humphreys, Jewell[1)] made an analysis of solar and arc wavelength displacements, and concluded that his measurements supported the existence of pressures between 2 and 7 atmospheres in the solar "reversing layer" where the Fraunhofer lines are produced. Jewell ignored the possible

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1) Jewell, Mohler and Humphreys; Ap.J., **3**, 138, 1896.

contribution of Doppler effects towards the observed displacements, since the latter were not directly proportional to wavelength and, in general, the amount of the shifts appeared to increase with the intensity of the lines. The figures were not very reliable, since there was no guarantee that the errors suspected by Rowland had been successfully eliminated.

An important result emerging from Fabry and Buisson's work is that pressure could not be regarded as the sole cause responsible for the production of the solar red shifts; for if it were, the asymmetrical broadening found by increasing the pressure in the arc lines should also be observed in the absorption lines of the solar spectrum. Such, however, is not the case. Furthermore, the appearance of absorption lines corresponds more closely to that of the spectrum produced in the vacuum arc and, as Evershed [1] was first to remark, this similarity in appearance favours a lower pressure (below atmospheric). Evershed maintained that these observed characteristics are much more convincing than the unjustified assumption that pressure is primarily responsible for the soler red shifts, as Jewell had supposed. Consequently, he advocated that the Doppler effett of localised radial convection currents yields the major contribution towards the

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1) Evershed, J.; Kod.Obs.,Bull.No.18, 1909.

measured values, while pressure exerts no appreciable influence on solar wavelengths. This opinion was not accepted by the majority of astronomers, however, because of the discoveries of the limb effect and its characteristics by Halm and Adams, which seemed to indicate that pressure did, in fact, play an important role in the production of spectral line shifts.

On the other hand, the behaviour of the cyanogen (CN) lines in Adams' observations suggested that the Doppler effect was also contributing towards the observed displacements. Laboratory experiments by Humphrey[1)] and by Rossi[2)] had shown that lines in the CN bands around 3883A and 4216A are quite insensitive to changes of pressure; yet the limb-centre shifts of those lines were not zero, as was expected. Since their relative wavelength shifts never exceeded 2 mA - equivalent to a line-of-sight velocity of about 0.15 km/sec. - Adams felt that this difference was quite compatible with Evershed's radial-current hypothesis; he attributed it to ascending radial currents at the Sun's centre producing a violet displacement relative to the wavelengths at its limb, where these currents (being at right-angles to the line-of-sight) would produce no effect. At this stage it was generally

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1) Humphreys; Ap.J., 6, 169, 1897.
2) Rossi, R.; Proc.Royal Soc.(A), 82, 518, 1909.

believed that the displacements of the solar lines could be completely explained by the super-position of Doppler and pressure effects, but further research by Evershed[1] and by Royds[2] indicated that the pressure in the outer layers of the Sun was approximately atmospheric, and thus supported Evershed's original contention that pressure is an insignificant cause producing the centre shifts. The Kodaikanal observers therefore attributed the limb increase to the Doppler effect alone.

At that time (1914), it was generally believed that a necessary consequence of this interpretation was that the centre-limb wavelength variation of all lines should follow a cosine law, due to the geometrical projection of the postulated radial currents when viewed at different angles to the line-of-sight; however, this was not found to be the case for the stronger lines included in Evershed and Royds' limb-effect observations, mentioned in the previous section. On the other hand, Halm's hypothesis (that relative differences in path length increase with increasing depth, especially near to the edge of the disk) would suggest that, if the pressure effect were producing the limb displacements, the weak lines should exhibit weaker (limb-centre) shifts, the

1) Evershed, J.; Kod.Obs.,Bull.No.36, 1913.
2) Royds, T.; Kod.Obs.,Bull.No.38, 1914.

opposite of what was actually observed. Another consequence of the Doppler current interpretation is that Sun-arc values at the limb should be zero, or at least very small; so in order to test its validity Evershed and Royds[1] made limb-centre comparisons of FeI wavelengths and added the values to the corresponding centre-arc shifts previously obtained. The resulting limb-arc displacements, however, were very much larger than anticipated and were quite incompatible with the radial current hypothesis. Yet neither did they support the belief that pressure is the major factor responsible for their production, for they exhibited no variation with wavelength nor dependency on line intensity. St. John, realising the inadequacy of Evershed's equipment, undertook a similar programme at Mount Wilson[2], taking special precautions to avoid sources of systematic observational errors, but the results of his extremely careful investigation merely served to confirm the conclusions of Evershed and Royds.

Evershed was unable to suggest any rational explanation of the large values obtained, and the only conclusion he could draw was that the Earth had some kind of repelling action on all lines coming from the Sun. This rather absurd

1) Evershed, J. and Royds, T.; Kod.Obs.,Bull.No.39, 1914.
2) St.John, C.E.; Ap.J., 46, 249, 1917.

hypothesis found no support from observations of Venus[2]. It seemed that the only way in which the situation could be satisfactorily resolved was by the introduction of another cause besides pressure and the Doppler effect that was capable of producing shifts of the order required; for errors due to pressure and pole-effects in the arc spectra were unlikely to be large enough to bridge the gap between the observed and required results, certainly in St. John's work. Happily, there already existed a well-known theoretical prediction which appeared to fulfil this requirement.

5. The Einstein Effect and the Observed Displacements.

In 1911, Einstein[1] derived from his equivalence relation that the wavelength (λ) of every line in the solar spectrum should suffer a displacement ($\Delta\lambda$) to the red end of the spectrum relative to the corresponding lines produced in a terrestrial light-source, the amount of which is given by the formula:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta V}{c^2} = 2.12 \times 10^{-6}$$

where ΔV denotes the difference in gravitational potential between the Sun and Earth's surface; and c is the velocity of light.

1) Einstein, A.; Ann.de Phys. (4), 35, 905, 1911.
2) Evershed, J.; Observatory, 42, 51, 1919.

All spectral lines are produced in a relatively thin layer of atmosphere at the same distance from the Sun's centre, hence the quantity ΔV will be constant for all lines of all elements at any chosen position on the solar disk; this implies that the ratio $\frac{\Delta\lambda}{\lambda}$ possesses similar characteristics, which should be reflected in the observations of Sun-arc displacements if the theoretical considerations from which the relation is derived are valid.

The first person to recognise in the observations of the solar red shifts a possible verification of Einstein's prediction was Freundlich[1], who noticed that the results of Fabry and Buisson for iron lines at the centre of the Sun's disk, and those of Evershed, corresponded very closely with the predicted values. He was also aware, however, that Evershed, Royds and St. John had clearly shown that the shifts of iron lines varied with intensity by an amount too large to be accounted for by differential pressure effects in the various layers of the solar atmosphere throughout which the spectral lines are formed. In addition, measurements by Royds (exempt from pole-effect) had yielded similar results for nickel and titanium[2], and thus established Jewell's

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1) Freundlich, E.F.; Phys.Zs., 15, 369, 1914.
2) Royds, T.; Kod.Obs.Bull.No.53, 1916.

original discovery that the shifts varied from element to element: they also showed that the Sun-arc shifts were not directly proportional to wavelength, and that their values at the centre of the disk were generally smaller than Einstein's theory requires - facts which suggested that some other effect was producing anomalies in either, or both, solar and terrestrial wavelengths. The observed increase in all these displacements in going from the centre to the limb was a feature of the solar lines only which was independent of the existence of the Einstein effect. Consequently, Freundlich rightly hesitated to claim that the observations at the time constituted a decisive verification of the theory.

It was clearly recognised by all concerned with the problem that the Einstein effect was certainly not the only cause responsible for the displacements of the solar lines. The centre-limb variation seemed to be explained by the Doppler effect, while the observed excess of some of the shifts at the limb over the predicted value could be attributed to pressure - but only in the case of pressure-sensitive lines. The wavelengths of lines insensitive to pressure were assumed to be affected by yet another cause, anomalous dispersion (or abnormal refraction). The theory of anomalous dispersion was first introduced by the Dutch physicist Julius in 1904[1], and later modified and revised to account for the increasing number of observational facts which were steadily being assimilated[2]. Basically, it stated that among the radiations emitted by the photosphere, those whose wavelengths are very near to those of the characteristic radiations of the vapours in the solar atmosphere must suffer abnormal refraction: it is, in effect, an application of Rayleigh's formula for scattering.[*] Julius' theory met with violent opposition from several notable practical astrophysicists, in particular from St. John[3] and Royds[4], with the result that it was never allowed to dominate the problem in the same way as the pressure effect has done, although it did provide scope for discussion for many years. Our present knowledge of the constitution of the solar atmosphere entitles us to disregard it altogether.

6. Researches on the Cyanogen (CN) Lines.

The CN bands, around 3883A and 4216A, were known to be

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1) Julius, W.H.; Rev.gen.des Sciences, 15, 480, 1904.
2) Julius, W.H.; Archiv.Neerl., 15, 417, 1910; 1, 231, 1912; 4, 59, 150, 1917; 5, 116, 1918.
3) St.John, C.E.; Ap.J., 41, 28, 1915; 46, 249, 1917.
4) Royds, T.; Kod.Obs.,Bull.No.48, 1915.
*) For a detailed account of the theory of anomalous dispersion the reader is referred to a memoir by F. Croze (Ann. de Phys., 19, 93, 1923).

insensitive to pressure, uninfluenced by pole-effect, and exhibited only very slight displacements from centre to limb, which Adams attributed to the Doppler effect. Hence, since these effects appeared to be so small in comparison with the predicted gravitational displacement, which varies between 8 and 9 mA over the wavelength region in question, an accurate measurement of the shifts of these lines was thought to provide a means of testing whether the Einstein effect exists.

Unfortunately, however, there are several reasons why the necessary precision cannot be attained in practice. The most significant of these is that the CN lines appear in a region of the solar spectrum which is overcrowded with Fraunhofer lines, and very few can be completely resolved. Not only might there be blends and consequent asymmetries among the CN lines themselves, but also among CN lines and metallic lines: the superposition of a metallic line on a CN line is likely to have the greater effect in shifting the true position of the latter.

Evershed and Royds[1)] were among the first to investigate this problem but, although they found good agreement with Einstein's prediction, they realised that the results were

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1) Evershed, J. and Royds, T.; Kod.Obs.,Bull.No.39, 1914.

not at all reliable on account of the limitations of their equipment, and that certain anomalies would have to be accounted for on a quantitative basis before the Einstein effect could be invoked. K. Schwarzschild[1], by measuring Sun-arc displacements at five different disk positions from centre to limb, discovered that the observed values were always decidedly smaller than the gravitational red shift. His work was presented by Einstein at the Academy of Sciences in Berlin on 5th November, 1914, and it produced a profound impression on all those interested in the development of the General Theory of Relativity. Nevertheless, there remained the possibility that the accidental errors due to displacements of the apparatus were not completely eliminated in Schwarzschild's measures, and since the diameter of the solar image was only 28 mm, small systematic errors were also liable to be incurred through poor resolution. St.John[2] made an extremely careful investigation of the shifts of the CN lines at the centre of the disk, using the 60-foot tower telescope and a high-dispersion spectrograph, which are essential requirements for such work; the excellent "seeing" conditions at Mount Wilson were an additional advantage in affording accurate results. St. John carefully selected

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1) Schwarzschild, K.; S.B.d.Berliner Akad., 1201, 1914.
2) St.John, C.E.; Ap.J., <u>46</u>, 249, 1917.

only a few CN lines which he believed to be least affected by the presence of blends, and employed four different procedures in obtaining the Sun-arc shifts at both the centre and limb of the Sun. His results must therefore be deemed the most reliable, yet they are radically opposed to Einstein's prediction.

Later researches on the CN bands were taken up by Grebe and Bachem[1)] which had a negative result, but served to emphasise the great care required when one attempts to provide a quantitative estimate of solar red shifts. These two physicists worked with a primary solar image of only 12 mm diameter, enlarged to 50 mm, and a linear dispersion of 1 A/mm, which provided results considerably inferior to those obtained by St. John (who had used a 170 mm diameter image and a dispersion of between 0.4 and 0.6 A/mm). But, by comparing microphotometric tracings of the solar and arc lines used in their investigations, Grebe and Bachem realised that such extreme care had to be exercised in the choice of lines used in the analysis of the observations that even St. John's

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1) Grebe and Bachem; Ber.d.Dt.Phys.Ges., 21, 545, 1919.
" Zs.f.Phys., 1, 51, 1920; 2, 415, 1920.
Grebe, L.; Phys.Zs., 21, 662, 1920.
" Zs.f.Phys., 5, 105, 1921.

results were liable to contain serious systematic errors which could invalidate his conclusion.

All that can be concluded from these researches on the CN bands is that the evidence is too scarce and too unreliable to furnish a definite answer to the question of the existence of the Einstein effect. Further interferometric investigations by Pérot on CN, iron, and magnesium lines[1], again employing inferior equipment, served only to confirm this fact. At this stage in the development of the problem it became clear that any interpretation of the observed shifts must necessarily be dependent on very few lines indeed.

7. The Modern Approach to the Problem.

In summarising the work described in this chapter, we need only remark that the period from 1896 - 1920 was one of great improvement in the experimental techniques and equipment employed in attempts to obtain a satisfactory solution of the problem of explaining the causes responsible for the production of the solar red shifts: its importance lies in the consequent elimination of sources of observational error which had invalidated many of the earlier measurements. Although all attempts to provide an accurate quantitative estimate of the shifts had met with little success, many

1) Pérot, A.; C.R., 170, 988, 1920; 172, 578, 1921; 174, 933, 1922.

reasons for the difficulties in attaining the necessary accuracy had become apparent. All the investigations were notable for the complete lack of bias towards any pre-established view concerning the treatment of the results obtained. The year 1920 marks the beginning of a second phase in the development of the problem, typified by the approach of Grebe and Bachem which has persisted right up to the present time.

The feature of Grebe and Bachem's work which distinguishes it from the earlier researches is that the relativity effect is assumed to be implicit in the observed values, and the problem under consideration is not to decide whether this effect exists, but rather to explain why the measured displacements are smaller than the theoretical prediction demands. The discovery which exerted such an overwhelming influence on all interpretations of observations made during this latter period (i.e. since 1920) was the knowledge that the analysis of photographs taken during the total solar eclipse at Sobral on 29th May, 1919, had shown that the positions of stars in the star-field surrounding the Sun exhibited shifts which seemed to conform with another prediction based on the General Theory of Relativity. After this was announced, it was generally taken for granted that the gravitational displacement of solar lines must also exist; but as McCrea[1)]

1) McCrea, W.H.; Proc.Royal Irish Acad., 57, A, 173, 1956.

has pointed out, these two predictions depend not only upon the basic theory itself but also upon different postulates concerning the properties of radiation, and consequently one is not entitled to infer the existence of the gravitational red shift from the results of eclipse experiments; in any case, the latter favour a value for the light-deflection significantly higher than that of 1".75 at the Sun's limb deduced from Einstein's theoretical considerations[1]. Very recently, however, physical experiments relating to the nuclear resonance absorption of gamma rays in crystals at low temperatures[2,3,4,5,6] would appear to have already provided an independent confirmation of the validity of the adopted viewpoint, although more work will still be required before a decisive quantitative check on the Einstein effect is obtained.

At the same time as this initial change in attitude was occurring, Dr Megh Nad Saha introduced his Ionization Theory[7] which revolutionised current notions of the conditions

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1) Mikhailov, A.H.: Observatory, 79, 82, 1959.
2) Mossbauer, R.L.: Z.Physik 151, 124, 1958: Naturwiss. 45, 538, 1958: Z.Naturforsch. 14a, 211, 1959.
3) Craig, P.R., et al.: Phys.Rev.Lett. 3, 221, 1959.
4) Lee, L.L.Jr., et al.: Phys.Rev.Lett. 3, 223, 1959.
5) Cranshaw, T.E., et al.: Phys.Rev.Lett. 4, 163, 1960.
6) Hay, H.J., et al.: Phys.Rev.Lett. 4, 165, 1960.
7) Saha, M.N.: Phil.Mag., 40, 472, 809, 1920; 41, 267, 1921.

prevailing in the outer layers of the Sun. Saha's theory, in a very convincing way, accounted for most of the features of the solar spectrum which had baffled astrophysicists for many years, and it was therefore not long in becoming generally accepted. For instance, the differing appearance presented by the Fraunhofer lines was realised to depend basically upon "the varying response of the elements with regard to the stimulus existing in the sun", and not so much upon the abundances of these elements, as had previously been supposed. Saha defines the word "stimulus" to denote all physical agencies tending to make the atoms luminescent, but shows how a high temperature alone can account for this state. He did, however, believe that the abundance was a major factor in determining the marginal appearance of the lines; thus he thought that, when the number of atoms taking part in the production of a Fraunhofer line becomes appreciable, the line would show itself in the solar spectrum.

The difficulty in making a numerical assessment of this condition led Fowler and Milne[1)] to work with the maximum intensities of the lines, rather than concern themselves with the uncertain conditions governing their marginal appearances. By finding that MgII lines and H lines both

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1) Fowler and Milne, M.N.R.A.S., 83, 403, 1923.

attained their maximum intensities at approximately the same temperature (about 10,000°K.), Fowler and Milne were able to determine the second important parameter in Saha's formula: viz. the electron pressure, which they estimated at 10^{-4} atmospheres. Thus the reason for the similarity between solar and vacuum arc spectra was made apparent, and it was immediately realised that neither pressure nor anomalous dispersion in the solar atmosphere could possibly have any appreciable effect in influencing the amounts of the red shifts observed.

The situation which astronomers then felt compelled to accept is that these displacements resulted from the superposition of Doppler effects on the constant relativistic red shift: this is the basic framework into which most later attempts to explain the observed phenomena have been forced. It has been usggested by Lindholm[1)] and Spitzer[2)] that small wavelength displacements may also be associated with collisional broadening, so this should also be regarded as an additional cause likely to influence both solar and arc wavelengths - and we are aware from our discussion in section 2 of this chapter that some spectral lines may also

1) Lindholm, E.; Arkiv.f.Mat.Astr. och Fys. 28B, No.3, 1941.
2) Spitzer, L.; M.N. 110, 216, 1950.

be appreciably affected by the Stark effect. However, our incomplete knowledge of many of the physical parameters which have to be determined if such shifts are to be calculated, and the complexity of the theories themselves, have so far prevented a quantitative treatment of these latter effects. Yet it would appear from the foregoing discussion that, unless at least one other effect besides the Doppler and Einstein effects is invoked to explain the observations, no satisfactory solution of the problem will be obtained.

CHAPTER II.

THE RELATIVITY-RADIAL CURRENT HYPOTHESIS.

1. The Observational Basis for St. John's Hypothesis.

St. John[1)] thought he could provide an adequate explanation of the solar red shifts by retaining his earlier belief that level is the determinative condition in their production: he believed the dependency upon depth to be well-established by concordant results from spectroscopic observations of solar rotation, sun-spots, flash spectra, differences between spectra from near the limb and at the centre of the Sun's disk, and the progression of the limb-centre wavelength differences with level of excitation. Accordingly, he subtracted the Einstein effect from each of his observed values and attributed the positive and negative residuals to variations in the effect of radial currents at different levels in the "reversing layer" - not to pressure, as he

1) St. John, C.E.; Ap.J., **67**, 195, 1928.

had originally supposed[1]. This is the interpretation which is commonly known as St. John's Relativity-Radial Current Hypothesis.

The observational data on which St. John's discussion is based comprise measures of the absolute wavelengths of 1537 spectral lines at the centre, and 133 lines at the limb of the Sun, and their wavelengths in a vacuum arc source.[*] All the values of the shifts on which St.John's conclusions were based are the means of closely agreeing grating measurements by St. John and interferometric observations by Babcock collected over a series of years, initially obtained at Mount Wilson with the 60-foot tower telescope and 30-foot focus spectrograph, and later with the 150-foot telescope in conjunction with the 75-foot spectrograph. Thus the results are heterogeneous, and might for this reason be expected to show inconsistencies among the individual values: yet the internal agreement was so good that many of the terrestrial wavelengths have been adopted as secondary standards. On the other hand, a comparison of the FeI solar wavelengths used by St. John in his discussion with the solar standards adopted by the

1) St.John, C.E.; Ap.J., 38, 341, 1913.
*) Actually, an arc in air with pressure corrections, and a vacuum arc, were both employed at different stages.

I.A.U. reveals the presence of marked discrepancies, the origin of which lies in the fact that the former were not independent determinations, but a combination of preliminary data obtained by Babcock with some from other sources. For the purposes of his investigation, St. John changed the scale of Babcock's measurements from that of 1928 to that of 1922 (cf. Trans.I.A.U. IV, 64, 1932). This naturally raises doubts as to the reliability of the figures tabulated in St. John's paper; at this stage, however, we do not wish to question the values themselves, but rather St. John's interpretation of them.

In assessing the overall reliability of his conclusions, one must consider the validity of the procedure of grouping spectral lines. Basing results on groups of measurements has the advantage that both systematic and accidental errors are reduced; the former by an uncertain amount, the latter by an amount depending on the number of individual measures comprising the group. In addition, this procedure will also minimise the effect of blends, whether they be known or unknown, whereas the position of any particular line might be seriously affected by a neighbouring line and so rendered unsuitable for discussion. In this connection, it is relevant to remark that, as a result of a critical survey of the whole of St. John's material; Schröter found that not more than about 80 lines appear to be entirely free from blends, as judged by a comparison with the Utrecht Atlas[1]; while a systematic investigation of the rest of the material provided a convincing demonstration of the fact that lines with a violet blend exhibit a smaller red shift than those with a red blend[*]. In principle, therefore, it is desirable to group measurements of the solar red shifts; in practice, the selection of lines which will constitute each group presents some difficulty, for the production of every spectral line is governed by so many factors.

St. John classified the lines according to line intensity, his criteria for selection being similarity in excitation potential and pressure classification (which refers to their behaviour under variations of pressure in the terrestrial light-source). He considered intensity rather than depth when discussing his measurements, for the simple reason that an estimate could always be made of the intensity of every spectral line which appears on a photographic plate, while there existed only a few lines whose levels of origin

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1) Minnaert, M., Mulders, G.F.W., Houtgast, J.: Photometric Atlas of the Solar Spectrum.

*) These facts were mentioned by Dr Schröter in a private communication to Professor E. Finlay-Freundlich, dated November 29, 1955.

were considered to be well-known; moreover, many of the estimates of level had been made on irregular features and disturbed regions of the solar disk and could not be taken as representative of the solar atmosphere in its steady state. However, because of the complexity of spectral line formation, the manner in which intensity is related to depth inside the solar atmosphere differs from line to line, and (as St. John himself was aware) it does not follow that lines of equal intensity originate at similar levels. We now know that a much better defined measure of the effective thickness of the absorbing layer is given by the quantity $\frac{W_\lambda}{\lambda}$, where W_λ denotes the equivalent width of a spectral line at wavelength λ.

Let us now examine St. John's hypothesis on the basis of his measures of 497 FeI lines of pressure classes a and b - lines measurable in the arc with very high accuracy - on which the weight of his conclusions depends. The results of St. John's groupings of the observed centre-arc displacements of these lines, $\Delta\lambda(o)$, and their residuals δ ($= \Delta\lambda(o)_{observed} - \Delta\lambda_{relativity}$) are tabulated in Table 1, and the observed values are plotted in Fig.1, from which it would appear that at least two factors are operative in determining the centre-arc shifts: viz. line intensity and wavelength, in the sense that an increase in either of these quantities corresponds to an increase in the observed displacement. Since St. John regarded the more predominant feature, namely the dependency of the red shifts on line intensity, as being the result of the correlation between intensity and depth, the observed fact that weak and presumably low-lying lines exhibited small shifts at the Sun's centre could be accounted for by assuming the existence of strong radial convection currents; however, at that time (1928), he could offer no satisfactory explanation for red shifts greater than the gravitational red shifts which were observed in strong lines and in high-level lines such as the H and K lines of CaII.

Although the wavelength dependency in St. John's observations is a feature of the centre-arc shifts which he considered only roughly in his groupings, it is important to study its form in more detail with a view to deciding whether the observed shifts do indeed have the character of Doppler effects and exhibit a direct proportionality to wavelength. We have investigated this question by first dividing St. John's data into classes according to the value of the line strength $\frac{W_\lambda}{\lambda}$ for all those lines for which the equivalent width W_λ is given in Allen's tables[1], thereby taking

1) Allen, C.W.; Memoirs Comm.Solar Obs.1,(6),1934; 2,(1),1938.

Summary of St.John's Data on the Centre-Arc Shifts

of FeI Lines belonging to Classes a and b.

(cf. Ap.J. 67, 147, 1928.)

TABLE 1.

The Intensity Dependency in St.John's Observations.

Wavelength and Pressure Classification	Mean Rowland Intensity	No. of Lines (N)	Mean Wavelength λ(A)	Unit: 1×10^{-3}A Mean Observed Shift $\Delta\lambda$(o)	Residual Shift δ
Violet, b	13.6	34	3943	+11.0	+2.7
	6.2	33	3917	8.2	0.0
	5	42	3974	7.1	-1.3
	4	76	4026	6.8	-1.7
	3	95	4106	6.5	-2.2
	2	73	4219	6.3	-2.6
	1	42	4269	5.9	-3.1
Blue, a	13.7	15	3830	11.3	+3.2
	5	31	4856	9.6	-0.7
	2.6	14	4629	6.6	-3.2
Red, b	6	23	6295	10.7	-2.6
	3	19	6311	9.7	-3.7

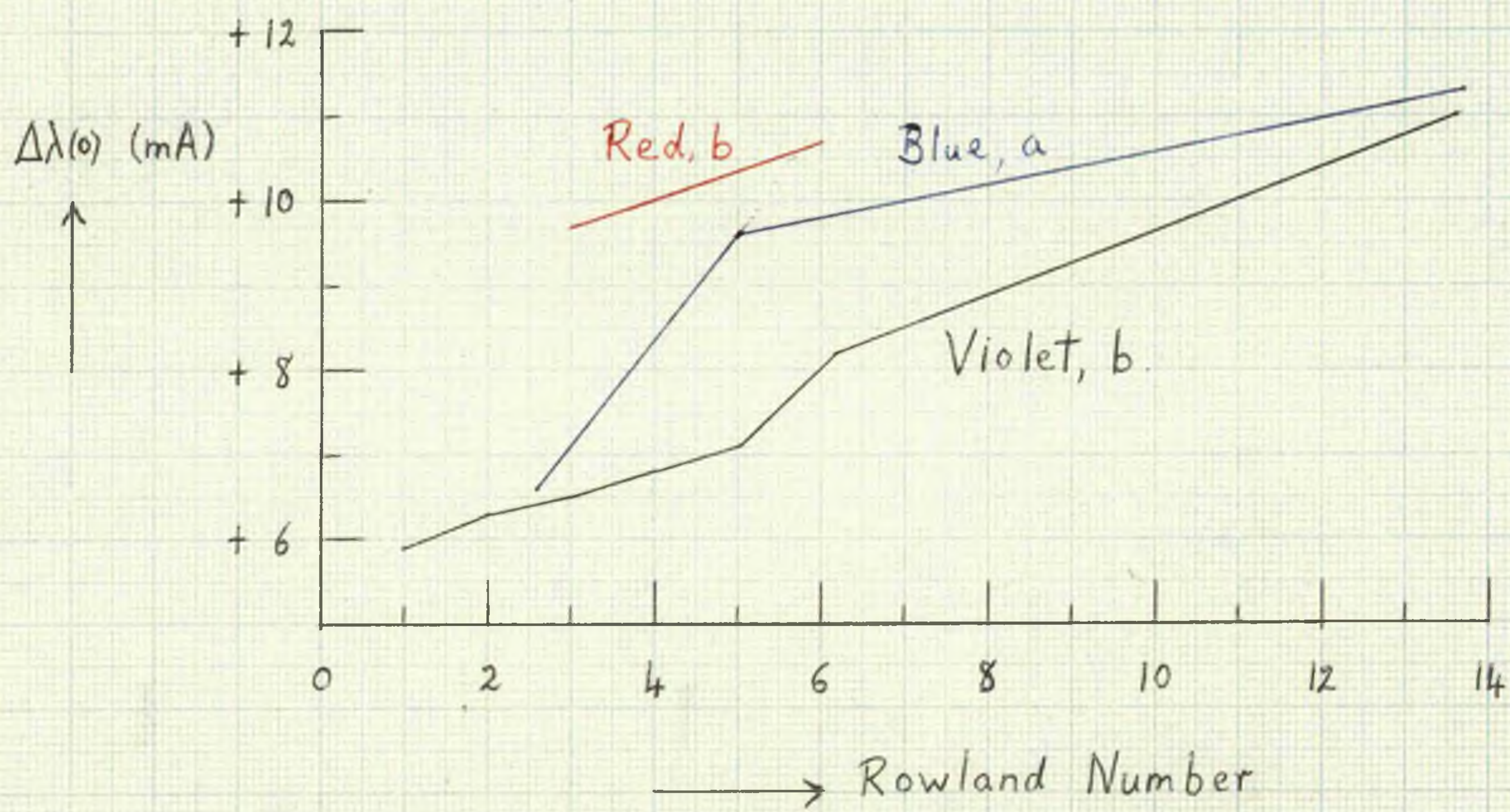


FIG.1: Intensity Dependency in St.John's Centre-Arc Shifts of FeI Lines.

account of the intensity dependence; then the lines belonging to each class and their observed centre-arc displacements were grouped according to wavelength. The results of this procedure are contained in Table 2. In accordance with St. John's hypothesis, we base our analysis upon the tabulated residual shifts since these are supposed to represent the Doppler effects of the postulated radial currents; accordingly, the values of this quantity for each intensity class, weighted by the number of lines comprising the group, have been fitted to the conditional equation $\delta = m\lambda$ by the method of least-squares and the results summarised in Table 3, along with the values of $\frac{\Delta\lambda(o)}{\lambda}$ derived from the relation:

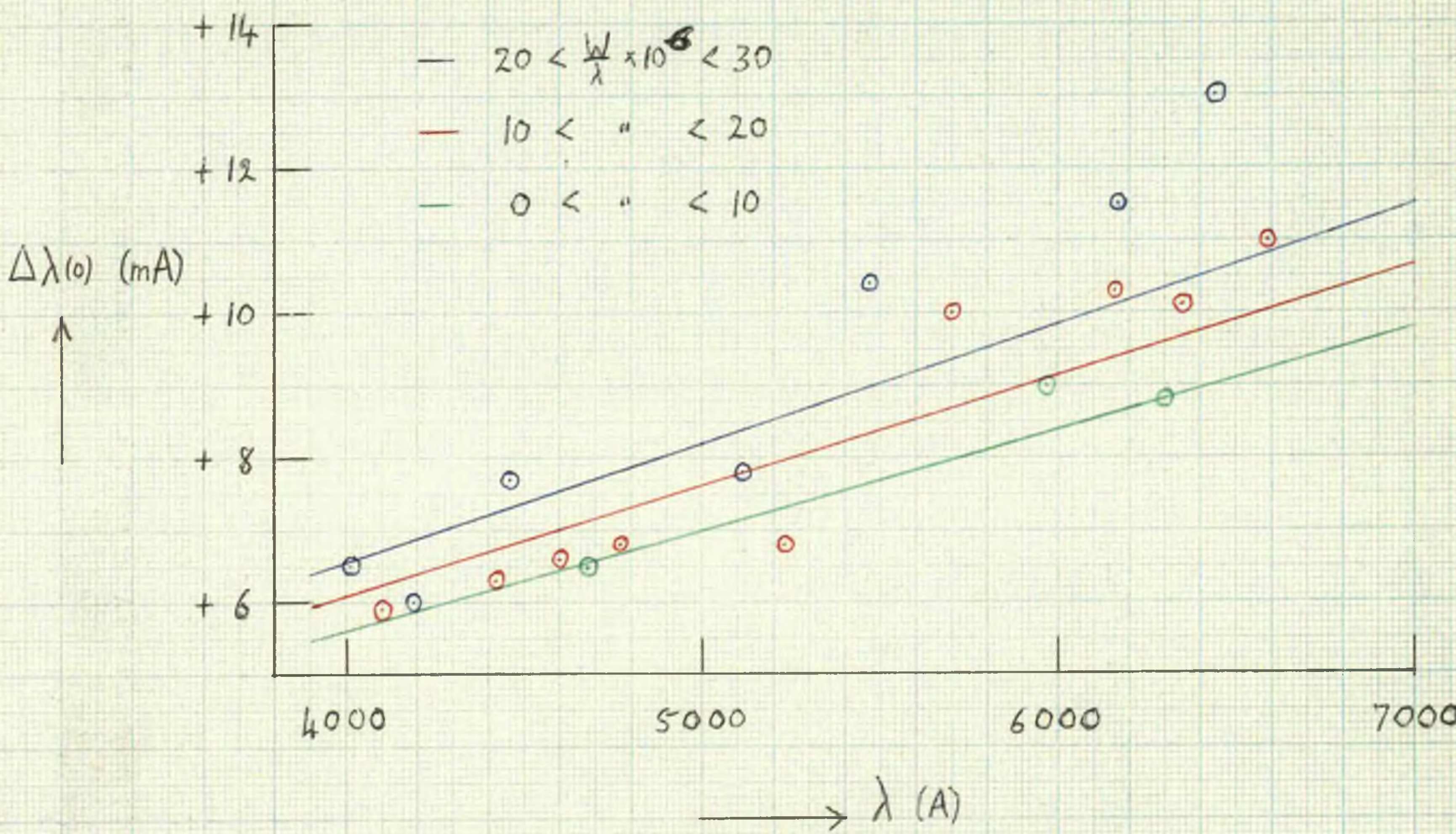
$$\frac{\Delta\lambda(o)}{\lambda} = \frac{\Delta\lambda_{rel}}{\lambda} + \frac{\delta}{\lambda} = 2.12 \times 10^{-6} + m$$

The quality of the representation of the observed values can be judged from Fig.2, paying due regard to the weight of each mean value (defined as the number of lines on which it was based): it would appear that a simple linear dependency provides an adequate fit of the data under discussion, bearing in mind the heterogeneity of St. John's measurements and the scatter among the observed values which may be produced by factors other than wavelength and line strength (e.g. level of excitation) which also affect the red shifts.

TABLE 2.

Dependency of St.John's Centre-Arc Shifts and Residuals of FeI Lines upon Wavelength and Line Strength (Group Values)

Line Strength $\frac{W}{\lambda} \times 10^6$	N	λ(A)	Unit: 1×10^{-3} A $\Delta\lambda$(o)	δ
> 40	16	4098	10.3	+1.6
	3	5405	11.0	-0.5
30 - 40	16	3991	6.8	-1.7
	7	4178	6.9	-2.0
	7	4460	9.6	+0.1
	3	5406	12.7	+1.2
20 - 30	34	4009	6.5	-2.0
	28	4185	6.0	-2.8
	14	4456	7.7	-1.8
	13	5108	7.8	-3.0
	5	5468	10.4	-1.2
	6	6169	11.5	-1.6
	2	6444	13.0	-0.7
10 - 20	25	4097	5.9	-1.8
	24	4406	6.3	-3.0
	14	4600	6.6	-3.2
	17	4773	6.8	-3.3
	9	5230	6.8	-4.3
	1	5702	10	-2.1
	7	6161	10.3	-2.8
	9	6345	10.1	-3.3
	1	6593	11	-3.0
0 - 10	16	4676	6.5	-3.4
	2	5966	9.0	-3.6
	11	6303	8.8	-4.6



⊙, ⊙, ⊙, are the group values corresponding to the respective representations.

FIG.2: Wavelength Dependency in St.John's Centre-Arc Shifts of FeI Lines.

TABLE 3.

Solutions of $\delta = m\lambda$ for St. John's
Centre-Arc Observations.
$\left(\frac{\Delta\lambda}{\lambda} = 2.12 \times 10^{-6} + m\right)$

N	$\frac{W}{\lambda} \times 10^6$	$m \times 10^6$	Probable Error $\Delta m \times 10^6$	$\frac{\Delta\lambda}{\lambda} \times 10^6$
19	> 40	+0.27	±0.13	+2.39
33	30 - 40	-0.24	±0.20	1.88
102	20 - 30	-0.48	±0.10	1.64
107	10 - 20	-0.60	±0.10	1.52
29	0 - 10	-0.72	±0.01	1.40

Thus the form of the wavelength dependency is compatible with St. John's hypothesis; hence we interpret the fact that m decreases numerically with increasing line strength as a consequence of the decrease of velocity with height inside the solar atmosphere, since we may suppose that in general - certainly for medium line strengths in the range 10 - 30 to which the bulk of St. John's measures refer - the higher the level of formation the smaller the outward velocity[1].

The wavelength dependency of the limb-arc measures cannot be determined from St. John's data, as these refer to lines contained in the ultra-violet and green regions of the spectrum only. St. John's grouping of the limb-arc shifts indicates no clear relationship with line intensity, in accordance with his interpretation; for at the limb the effect of Doppler currents should vanish, leaving the Einstein effect which depends only upon wavelength and gravitational potential. However, the mean values of the limb-arc shifts are appreciably in excess of this limiting value - a discrepancy which St. John was unable to explain. In view of this difficulty, and the problem of accounting for the positive residual in the observed centre-arc shifts of very intense lines (cf. Table 1), it can scarcely be claimed that

1) cf. Adam, M.G.; M.N., 118, 110, 1958.

St. John's own treatment of his extensive observational material provided a satisfactory verification of the relativity-radial current hypothesis. Nevertheless, this interpretation became generally adopted, largely because it was felt that there existed no alternative.

2. Evidence for Anomalies in the Allegheny Observatory Red Shifts.

While St. John was busy accumulating the measurements of Sun-arc shifts on which he based his new interpretation, an extensive set of interferometric observations covering the visible spectrum was being made at the Allegheny Observatory in collaboration with the spectroscopic section of the Bureau of Standards (i.e. A.O. - B.S. observations). The characteristic observational feature of the A.O.-B.S. investigation was the pronounced intensity dependence in the values of the shifts. Since this phenomenon appeared to be associated with an asymmetrical broadening of the lines towards the red, the A.O.-B.S. observers regarded its existence as an insuperable obstacle to the acceptance of the predicted relativity effect[1] and were consequently unwilling to accept St. John's explanation. The experimental

1) Burns, K.; J.O.S.A., 20, 216, 1930.

arrangement employed by Burns, Meggers and Kiess for obtaining the A.O.-B.S. wavelengths was the following[1]:- An image of less than 4 mm in diameter formed by a telescope of 40 cm focal length was focussed upon one of the plates of a Fabry-Pérot interferometer, and the resulting system of interference fringes projected by a ring lens on to the slit of a powerful grating spectrograph which dispersed the light; thus, by "crossing" the two instruments, high resolution and dispersion were attained. The spectral lines so formed were photographed simultaneously alongside a neon comparison spectrum which yielded standard wavelengths whose values had been derived from a previous investigation carried out at the Bureau of Standards[2].

The reason for working with such a small solar image was twofold: one of the primary aims of the A.O.-B.S. investigation was to provide accurate wavelengths in the solar spectrum, for it was realised that various systematic errors were inherent in the values given by Rowland[3] which "no simple change can correct". Hence the A.O.-B.S. solar wavelengths were to be compared with those measured by Rowland, who had worked with a small solar image

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1) Burns, K. and Meggers, W.F.; Publ.Allegh.Obs.VI; No.7, 1925.
   Burns, K. Publ.Allegh.Obs.VI, Nos.8 and 9, 1927.
2) Burns,K.,Meggers,W.F.and Merrill,P.W.;Bull.Bur.Stds.14,765,
3) Rowland,H.; Preliminary Table of Solar Spectrum /1918
   Wavelengths, 1895(Carnegie Institute of Washington).

image[*], and so it was desirable, on principle, that the Allegheny observers should do likewise. The more compelling reason, however, was that in practice it would otherwise have been impossible at that time (1924-27) to obtain spectral line images of measurable intensity on many of the photographic plates unless the exposure times were of several hours' duration, due to so much light being lost in reflection and absorption by the interferometer, as well as at the slit and at the grating. Thus it was that the A.O.-B.S. wavelength measures were, by necessity, based upon the spectrum of integrated sunlight.

It has been suggested by E. Lau[1] that the entire intensity dependence in the red shifts found from these observations could be accounted for through the use of the Fabry-Pérot interferometer, but Burns replied that he had estimated a mean wavelength shift due to the interferometer of only 0.3 mA under the actual conditions of observation. The belief that this is not the cause of the intensity dependence in the A.O.-B.S. investigation is supported by the lack of an intensity equation between the A.O.-B.S.

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*) The writer has been unable to find any reference to the actual size of the image employed by Rowland, but in Trans.I.A.U.,VI,63,1938, it is explicitly stated that Rowland's spectrograms are of "more or less integrated sunlight".

1) Lau, E.; Phys.Zs. 27, 870, 1926.

and Rowland's wavelength measures, which Burns and Meggers felt was "a most encouraging feature of the comparison". Sinee Rowland did not employ an interferometer, it follows as a consequence of the general systematic increase in wavelength towards the limb (i.e. the limb effect) that the procedure of projecting light from all regions of the disk indiscriminately upon the plate of the intererometer, and not the effect of the interferometer itself, is responsible for the asymmetrical broadening towards the red and its associated intensity dependence shown by the A.O.-B.S. wavelengths. Another source of asymmetry is the presence of blends, which would be most likely to cause serious errors in the violet region of the spectrum, where the spectral line density is high.

In order to assess the overall effect of the anomalies thus produced in the A.O.-B.S. measures we have subjected this material to an analysis similar to that already carried out on St. John's data; the results are contained in Table 4*), and the most significant of these (for $\frac{W_\lambda}{\lambda}$ between 10 and 30) have been plotted in Fig.3. The comparison with the corresponding representations obtained from St. John's data provides a striking illustration of the systematic

*) Lines of weight D, and those listed as blends in the Revised Rowland Table(1928), were omitted from our analysis.

character of the anomalies in the line strength and wavelength dependencies found in the A.O.-B.S. red shifts. We do not propose, at present, to venture a quantitative explanation of these features, since it is difficult to decide the extent to which they are influenced by the procedure of employing integrated sunlight, and by systematic observational errors associated with the interferometric method of parabolic channels[1]: this question will be considered later. We do, however, wish to stress that, in view of the results of this discussion, the A.O.-B.S. solar wavelengths should neither be employed as standards nor as a basis for examining the line strength or wavelength variation of the Sun-arc shifts.

On principle, this conclusion implies that no great significance should be attached to the values of the correlation co-efficients between these displacements and the quantity $\frac{W_\lambda}{\lambda}$ found by Miss Adam[2] from a discussion of the results of the A.O.-B.S. observations and her own interferometric measurements made at Oxford. Adam herself has explicitly stated that a quantitative agreement between the A.O.-B.S. and the Oxford results is not to be expected on account of the differences known to exist between integrated-disk

1) cf. Adam, M.G.; M.N., 112, 564-5, 1952.
2) Adam, M.G.; M.N., 115, 405, 1955.

TABLE 4.

Dependency of the A.O.-B.S. Red Shifts of FeI Lines upon Wavelength and Line Strength.

$\frac{W}{\lambda} \times 10^6$	N	λ(A)	$\overline{\Delta\lambda}$(mA)
> 40	7	4243	10.9
30 - 40	2	3964	5.5
	3	4220	7.0
	5	4448	12.6
	1	4860	10.0
20 - 30	10	4087	5.4
	7	4201	4.9
	6	4429	8.3
	4	4667	10.3
	2	4953	12.0
	10	5050	12.4
	1	5263	13.0
	2	5473	12.0
	4	6133	9.0
	4	6364	6.3
10 - 20	2	4327	10.5
	4	4629	9.3
	7	4858	11.4
	3	5077	12.0
	6	5281	11.2
	11	5941	6.0
	8	6110	4.1
	15	6333	5.7
	3	6585	9.0
0 - 10	3	5985	9.0
	2	6439	5.0

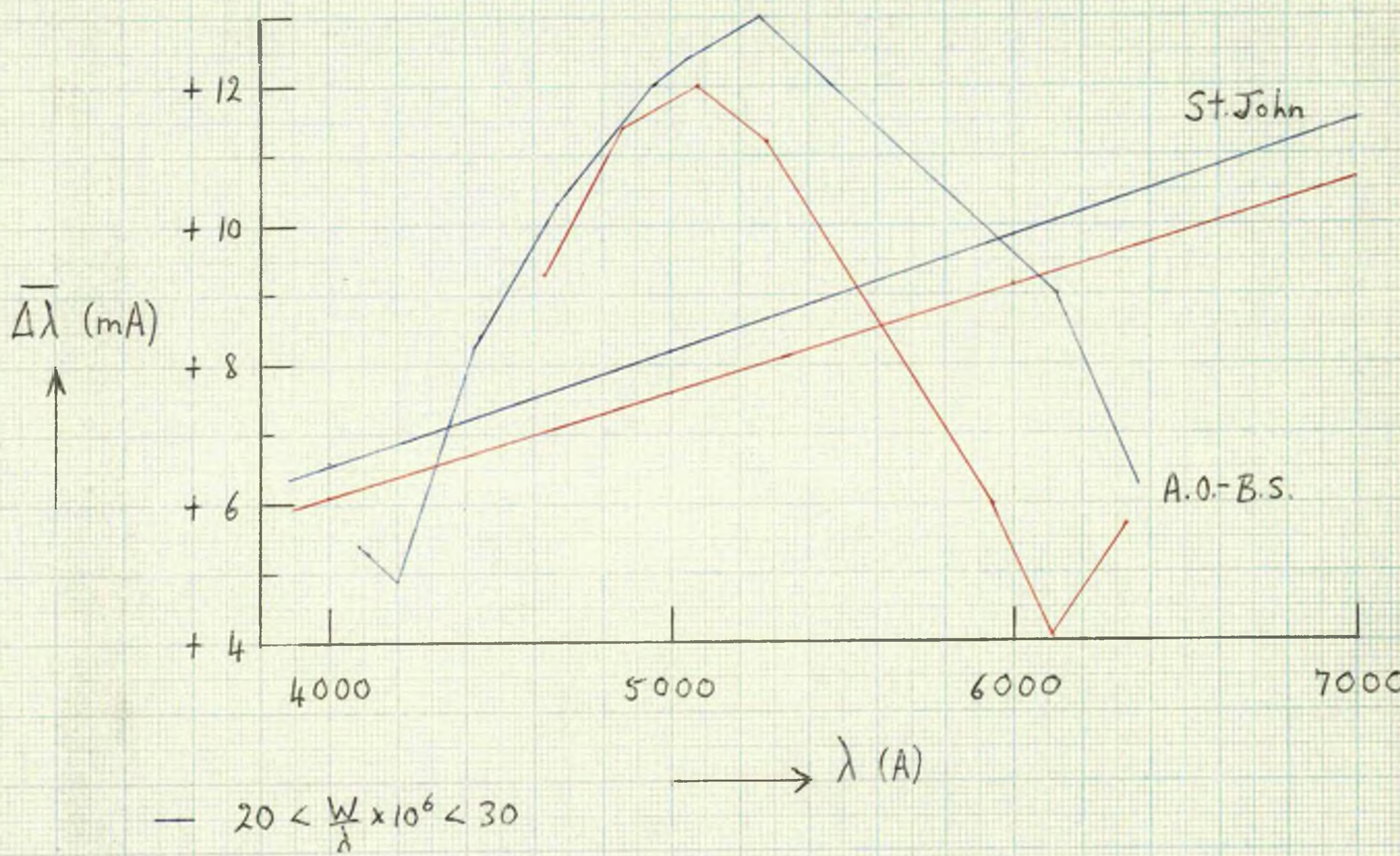

FIG.3: Comparison of A.O.-B.S. and St. John's Results for FeI Lines.

wavelengths and centre-of-disk values. A further corollary to the foregoing discussion is that the A.O.-B.S. measures cannot be used to test St. John's hypothesis, nor do they constitute any evidence against it.

3. The Problem of the Limb Excess.

A serious difficulty against accepting St. John's hypothesis was soon to come from measurements of the limb-arc shifts which yielded values considerably in excess of the relativity effect at the Sun's limb, where the Doppler effects of radial gaseous streaming must be negligibly small. From observations of the H and K lines of CaII in 200 prominence spectra obtained at Ewhurst, Evershed[1] found that the relativistic red shift was exceeded. The exact amount of this discrepancy was uncertain, due to the conditions in the arc, and the fact that the effects of solar rotation may have been wrongly assessed because of the assumption that the red shifts not due to rotation are Doppler effects occurring at random[2]. Although the latter difficulty is inherent in all measurements of this nature and cannot be avoided, Evershed felt that more reliable values for the red shifts would be obtained if he were to

1) Evershed, J.; M.N., 88, 126, 1928; 89, 250, 1929.
2) " M.N., 85, 608, 1925.

compare the solar H and K lines directly with a carbon arc in which these lines are prominent, rather than measure their wavelengths indirectly against the 1922 I.A.U. iron standards, which he rightly suspected were still in need of correction[*]. Thus, when obtaining the shifts of the H and K lines in the reversing layer[1], Evershed used a Pfund arc in air, operating at 5 amps and 100 volts (which very nearly satisfies the I.A.U. specifications) with carbon in between two mild steel pole-pieces. As a result of direct measurements, after the addition of a small pressure correction of 1.7 mA, he found a mean centre arc displacement of +20.7 mA for these two lines; however, spectra obtained with light derived from a region 5" to 10" within the limb yielded a mean limb-arc shift of only +15.1 mA, thus indicating a decrease in wavelength from centre to limb.

The latter value had also been found by Evershed from previous observations of 380 prominence spectra at various points just <u>outside</u> the limb of the Sun, where light does not traverse any portion of the chromosphere, and the lines are not broadened as they are in limb spectra. The analysis and comparison of all data collected during the entire period

*) It so happens, however, that the correction required in this spectral region is small (only -1 mA).

1) Evershed, J.; M.N., 91, 260, 1931.

from August 1926 to December 1934 indicated that this figure was probably too large by 1.5 mA[1]; but when considering the reality of such a small difference it is important to record that from the analysis of all 782 spectra Evershed discovered a latitude effect in his values which he believed might be related to sun-spot disturbances: between the solar equator and latitude 30^{0}, the general red shift of the H and K lines in the prominences was approximately 14.6 mA, whereas at latitude 40^{0} (outside the zone of sun-spot disturbances) it dropped to 10.5 mA. In addition, there is always the possibility that these statistical results may be affected by small selection errors; consequently, Evershed based his final conclusion on the entire prominence data and states: "The shift of H and K may be said to exceed the relativity shift by not less than .005 and not more than .0065 A".[2].

Precise interferometric measufements by C.V. Jackson of the H and K vacuum arc wavelengths[3], when subtracted from the solar wavelengths given in the Revised Rowland Table (corrected to the 1928 standards), yielded Sun-arc shifts of 19 mA and 23 mA for the H and K lines respectively[†]; and

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1) Evershed, J.: M.N., 95, 503, 1935.
2) " " " " " .
3) Jackson, C.V.: Proc.Royal Soc.A., 130, 395, 1931.
†) Actually, there is a misprint in Jackson's paper: the shifts quoted for the H and K lines ought to be interchanged.

when substituted for the arc wavelengths of St. John[1] on which Evershed had originally based his earlier measures of the wavelengths of these lines in solar prominences[2], they gave a resulting mean value of +14 mA. These figures are in complete agreement with Evershed's results[3], and would appear to confirm the reality of the decrease in shift between centre and limb as well as the excess of 5 mA over the relativity shift at the edge of the disk.

Not only the H and K lines exhibited an excess shift, however. Evershed had also made measurements of the limb-arc and centre-arc shifts of two strong Al I lines in the H and K region of the spectrum, which yielded mean values slightly in excess of the Einstein shift, but no indication of a centre-limb wavelength variation. Similar results also applied to the Na -D lines: earlier measures of these lines had failed to provide evidence of such a variation[4] and from a re-determination of the Sun[*]-arc shift of the D line only, Evershed found a value of +13.4 mA, which is less than 1 mA in excess of the relativity shift in this part of the spectrum[5].

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1) St.John, C.E.; Ap.J., <u>31</u>, 156, 1910.
2) Evershed, J.; M.N., <u>90</u>, 189, 1929.
3) Jackson, C.V.; M.N., <u>93</u>, 98, 1932.
4) Evershed, J.; Observatory, <u>46</u>, 303, 1923.
*) In view of the previous result, no distinction was made between light from centre and limb.
5) Evershed, J.; M.N., <u>98</u>, 195, 1937.

But the largest excess was found for the H_α line, which despite its breadth is still measurable with a high degree of consistency. A comparison between Curtis's value of the vacuum arc wavelength of this line[1] and the solar wavelength given in the Revision of Rowland's Table, yields a centre-arc shift of +23 mA: the limb-centre difference measured by Adams was +2 mA, hence the limb-arc shift amount to +25 mA, an excess of +11 mA over the Einstein effect!

Along with the measures of the H and K lines and the Al I lines in the reversing layer, Evershed published the results of a large number of measurements on 22 Fe I lines in the same spectral region[2], derived from spectra collected at Kodaikanal (from 1912 - 22) and at Ewhurst (from 1929 - 30). By grouping these wavelengths according to intensity, he found limb-arc shifts of +15.5 mA and +13.8 mA for the strong and weak lines respectively: later measurements on 7 strong and 8 weak lines in the H and K region (photographed both at the poles and at low latitudes) yielded a mean limb displacement of +14.6 mA, confirming the former set of results[3]. Thus the FeI lines exhibit a similar excess as the H and K lines at the edge of the Sun; the essential difference

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1) Curtis, W.E.; Proc.Royal Soc.A., 90, 605, 1914.
2) Evershed, J.; M.N., 91, 260, 1931.
3) " " 95, 503, 1935.

between the two sets of results lies in the fact that the FeI wavelengths increase towards the limb, whereas the CaII wavelengths decrease.

The significant features of these researches by Evershed are the pronounced intensity dependence of the centre-arc shifts which gave rise to over-relativistic values for the stronger lines measured (i.e. those of intensity 7 and over), and the large limb-arc shifts which were found in all cases to exceed the Einstein value. Evershed held the opinion that both of these observed effects were direct consequences of using a small solar image (of about 60 mm diameter), due to scattered light in the case of the centre-arc values, and a combination of this with the low telescopic resolution at the limb position: he attributed the large scatter found among the individual plate values to the existence of localised radial currents and to irregularities in the rotation shift at the centre and limb positions respectively. In order to test his belief, Evershed next measured the limb-arc shifts of 5 strong FeI lines in the red region of the spectrum[1], where any effect of atmospheric scattering would be much less, and obtained the surprising result that the mean observed shift was practically twice the Einstein effect! He checked

1) Evershed, J.; M.N., 96, 152, 1936.

this by adding the limb-centre differences of Adams[1] to the centre-arc shifts of St. John[2] for 4 of these lines, and these indirectly obtained values yielded a mean limb-arc shift of +25 mA, precisely the same as that found from his own direct measurements - and, incidentally, the same as that which he deduced for the H_α line.

We have already pointed out that the wavelength dependency of the limb-arc shift (or of the excess above the Einstein effect) cannot be accurately determined on the basis of St. John's data, since his limb-arc measures refer only to lines contained in the ultra-violet and green regions of the spectrum (cf. II,1). In order to supplement these values in the violet, blue and red, Evershed had made direct observations of the limb-arc shifts on spectra obtained both at Kodaikanal and at Ewhurst and - after applying the appropriate pressure corrections - he compared these with the corresponding shifts of 105 FeI lines for which the centre-arc displacements had been measured by St. John and the limb-centre values were available from previous researches carried out at Mount Wilson and at Kodaikanal[3]. This comparison confirmed that the close agreement found in the red between the directly and

1) Adams, W.S.; Ap.J., 31, 45, 1910.
2) St.John, C.E.; Ap.J., 67, 141-2, 1928.
3) Evershed, J.; and Royds, T.; Kod.Obs.Bull.No.39, 1914.
Aayar, A. Kod.Obs.Bull.No.44, 1915.

indirectly obtained shifts also applied to the blue and violet regions of the spectrum; furthermore, in the green and ultra-violet, St. John's own direct measurements of the limb-arc shifts were found to be verified. In this way it was established for the whole of the visible range that a large excess shift is characteristic of measurements made by different observers. Since in one case the observations were made with the large solar images of the Mount Wilson solar towers, this feature cannot be a consequence of employing a small solar image, as Evershed had originally believed. Furthermore, although the wavelength variation in the excess shift was found to be irregular, the general tendency is for the values to increase towards the red, which is opposed to the explanation that the limb excess is an effect of scattered light.

With regard to the centre-arc displacements, Evershed states[1)]: "My own measures of solar and arc lines in general sunlight and at the centre of the Sun's disc are in good agreement with those of St. John", from which we may infer that the same intensity dependence characterises both sets of observations; consequently, the results obtained by Evershed and his collaborators are not appreciably influenced

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1) Evershed, J.; M.N., 96, 152, 1936.

by anomalies of the type found from our comparison of the A.O.-B.S. and St. John's data. Indeed, Evershed stated on two occasions[1] that, in contradiction to Burns and Meggers' results, his limb spectra showed perfect symmetry, and no indication of the broadening on the red edge. Since this condition is necessary for the acceptance of St. John's interpretation, Evershed's opinion then (1937) was that "there can be little doubt that the Einstein effect accounts for most of the shift in the solar spectrum". The limb excess, however, has remained an unsolved problem.

4. The Limb Effect and St. John's Hypothesis.

A weakness in St. John's investigation was the omission of intermediate measures across the disk which on account of the variation of optical depth with disk position are capable of providing a quantitative check on the depth-dependency of the Doppler currents derived from the centre-arc displacements on the basis of the relativity-radial current hypothesis. The pioneer observations by Halm and by Schwarzschild (CN bands) are neither extensive nor accurate enough for this purpose, but some information was obtained from the investigation by Evershed and Royds (1916) described in Ch.I,3; however, the poor resolution afforded by the small solar

1) Evershed, J.; M.N., 91, 260, 1931; Observatory 60, 266,1937.

image employed (14mm diameter, enlarged by a factor of two) limits the significance which may be placed on these results. A feature of the limb effect which first came to light in the reduction of these observations was that the relative shifts exhibited no appreciable dependency upon the solar latitude at which they were observed. This discovery was later confirmed by Freundlich, v.Brunn and Brück[1] employing superior equipment at Potsdam, comparable in resolution and dispersion with that used by St. John.

The Potsdam investigation was restricted to 9 FeI lines with wavelengths between 4347A and 4490A (mean wavelength 4426A), belonging to the same multiplet $^5D - {}^7F^o$, which were selected on the criteria of symmetry[*] and sharpness of appearance in both solar and vacuum arc spectra. The limb-centre wavelength displacements were measured at 6 positions along 12 different radii symmetrically disposed over the area of the Sun's disk. After carefully applying corrections for the effect of solar rotation, Freundlich and his collaborators found such excellent agreement among the values of

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1) Freundlich, E., v.Brunn, A.and Bruck,H.; Zeits.f.Ap.,1,43,193

*) An examination of the microphotometric tracings of these lines in the Utrecht Atlas shows that to some extent all 9 lines are affected by blends which had not been apparent on visual inspection; however, it is thought to be unlikely that the mean shifts resulting from this investigation are appreciably affected by this cause.

the relative shifts of all 9 lines at every disk position, that they based their final results upon the mean values. The fact that the shifts of the different lines were found to be equal implies that the cause (or causes) responsible for their production is effective to the same extent in lines belonging to the same multiplet, although the generality of this conclusion must now be held in question, in view of Luise Herzberg's discovery that systematic variations can occur among the shifts of lines belonging to the same multiplet.[1)] The fact that the variation of the shift across the disk was identical for all radii suggests that the Sun is symmetrical with regard to this cause, so that it should be sufficient in all investigations of the limb effect to confine the observations to one radius only.

The mean limb increase found from the Potsdam investigation bears no similarity to a cosine curve: rather a secant law provides an accurate representation of the observations. The Potsdam observers conclude from this fact that "it is as impossible to explain the effect by radial streaming as by simple Compton scattering." The same belief was also held by Hunter, who discussed the existing observations of the limb effect and limb-centre differences and came to the conclusion

1) Herzberg, L.; Canadian Journal of Physics, 35, 766, 1957.

that "no hypothesis yet put forward will account for all the facts"[1]. This view, however, was abandoned that same year (1934) when McCrea and Mitra[2] pointed out that if there is to be no accumulation of matter at any level, the postulated radial velocity should not be supposed constant, but should decrease towards the centre of the Sun's disk, where one observes the deepest atmospheric layers (i.e. the regions of greatest density). This important consideration leads to a modification of the simple cosine law in keeping with the observed limb effect. McCrea and Mitra demonstrated that quite a good representation of the Potsdam measures can be obtained by assuming that the density gradient in the solar atmosphere is adiabatic: the agremment was less satisfactory in the case of Evershed and Royds' observations, but these carry much less weight. Thus it would appear that St. John's hypothesis is quite compatible with the observed form of the limb increase, although these relative observations are in themselves insufficient to provide a complete test of this interpretation. Centre-arc values of the shifts must also be available in order to confirm that the radial velocities deduced from the limb effect observations do in fact correspond to those derived from the absolute measures.

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1) Hunter, A.; M.N., 94, 594, 1934.
2) McCrea, W.H. and Mitra, K.K.; Observatory, 57, 379, 1934.

This dual requirement was fulfilled by a set of interferometric observations by Miss Adam[1] of the centre-arc shifts and limb-arc shifts at 6 different disk positions along the polar diameter, for 14 Fraunhofer lines in the spectral region around 6100A which were specially selected for discussion on the criterion of apparent freedom from blends and asymmetries, as judged by reference to the Utrecht Atlas. The wavelengths of these lines were based on neighbouring telluric lines, although neon comparison spectra were obtained at intervals during a sequence of solar exposures, and yielded a valuable confirmation of the stability of the apparatus. A statistical analysis of the entire observational material, which Miss Adam kindly placed at our disposal, has shown that there was a slight tendency for the internal accuracy to improve with increasing line width, and confirmed that the lines were measurable with the same degree of precision at all disk positions: the scatter of the individual plate values followed a normal distribution and yielded a probable error of less than $\pm$ 1 mA in the mean values tabulated in Adam's paper; the published centre-arc shifts being the means of 11 individual spectra, and the other the means of 6.

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1) Adam, M.G.; M.N., 108, 446, 1948.

Adam, of course, was fully aware that this was the order of accuracy obtainable, but since she found no evidence of systematic differences of behaviour across the disk with line strength, element, or centre-arc value, she decided that there was "no justification for more than a statistical treatment of the present material", and accordingly based her discussion upon the mean values obtained by averaging the shifts of all 14 lines at each disk position. A comparison between the resulting mean limb effect and that found from the Potsdam observations indicated that the two independent sets of relative values are in excellent agreement, thereby proving that there is no significant change in the form of the relation with time, or with wavelength region throughout the visible spectrum; furthermore, as Adam pointed out, neither did element nor line intensity seem to exert an appreciable influence on the limb effect of the individual lines included in her data.

Adam used this observational material to test St.John's relativity-radial current hypothesis, and found that this led to "results which are inconsistent with our knowledge of the solar spectrum and of the physical conditions prevailing in the solar atmosphere." This conclusion was based upon the negative result obtained from a consideration of the possibility that systematic radial velocities of about 1 km/sec. might be introduced by the solar granulation, due to the rising (hotter) gases radiating more energy than the down-going (cooler) ones, thereby producing a small asymmetry in the line profile; for this seemed to be the only plausible physical mechanism by which a steady state could be maintained. There are, however, several reasons why this result needs revision:-

a) By averaging out indiscriminately absolute displacements which spread over a range of about 10 mA at each disk position, Adam was implicitly assuming that the variations from one line to another possessed the character of accidental errors: but further observations at Oxford of centre-arc shifts employing the interferometric method of circular channels have since cast doubts upon the validity of such a treatment. The overwhelming advantages of this experimental technique, which was originally introduced by Treanor[1] in 1949 and subsequently developed at Oxford, over the older method of parabolic channels are that it affords a direct comparison of the cadmium standard with any region of the spectrum, and permits a simultaneous exposure for the Sun and standard source through the same optical train: it also provides a

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1) Treanor, P.J.; M.N., 109, 389, 1949.

higher internal accuracy among the measures, as the circular channels give much clearer impressions on the photographic plates. Its application to interferometric measurements of solar wavlengths[1], and the higher accuracy obtained from the increase in resolution, have resulted in the discovery that the set of absolute wavelength values under discussion contains appreciable systematic discrepancies due principally to scale errors in the secondary standards. These instrumental errors should have no effect on the relative differences, however, since all measured displacements of a particular spectral line ought to be affected in precisely the same way.

b) The mean velocity ($\bar{v}$) used to determine the absolute values of the granular streaming was obtained from the observations after allowing for the relativistic red shift (in accordance with St. John's hypothesis) and correcting for the Lindholm effect[2]. However, in calculating the values of the Lindholm shift from the damping constants determined by ten Bruggencate and Houtgast[3], Adam made no allowance for the fact that

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1) Adam, M.G.; M.N., 112, 149, 1952; 115, 422, 1955.
2) Lindholm, E.; Arkiv.f.Mat.,Astr.och Fys. 28B, No.3, 1941.
3) ten Bruggencate, P. and Houtgast, J.; Zeits.f.Ap., 20, 149, 1941.

the wavelength region to which her own red shift observations refer (around 6100A) differs from that for which the values of the damping constant had been obtained (5100A). When this is taken into account, one finds that the correction to the mean centre-arc displacement is only 2.0 mA, as compared with the corresponding value of 4.1 mA deduced by Adam: the absolute corrections become decreasingly smaller as the limb is approached. Consequently, the Lindholm effect can have only a minor influence upon the form of the function representing the limb effect of medium-intense solar lines. This, together with the fact that there exists at present little independent observational evidence to support Lindholm's theory, suggests that we ought to ignore it altogether as a significant cause contributing to the production of the solar red shifts.

c) The value adopted by Adam for the intensity ratio between granula and intergranula: viz. $\frac{J_1}{J_2} = 1.10$ was based upon the 5" granulation observed by Plaskett[1]. Most later estimates, however, by Keenan[2], Thiessen[3], Rösch[4] and others agree in yielding a much lower value

1) Plaskett,H.H.: M.N., 96, 415, 1936.
2) Keenan, P.C.; Ap.J.; 88, 360, 1938: 89, 604, 1939.
3) Thiessen,G.;Zeits.f.Ap.,35,237,1955: Naturwiss.,37,427,1950.
4) Rosch,J.; C.R. 240, 1630, 1955: 243, 478, 1956.

of about 1" for the mean granule diameter: this latter figure is confirmed by the high-definition solar photographs from 80,000 ft. recently obtained by Schwarzschild, Rogerson and Evans[1] from unmanned balloon flights, which indicate that the size of the elements range from 2" down to 0".3, and at the same time provide a verification of the fact that the granulation has a cellular though highly irregular character. The intensity ratio is very difficult to assess accurately due mainly to the influence of scattered light and atmospheric scintillation, but most observers have favoured a value greater than 1.10.

When due allowance is made for the above facts, one finds that the granular velocities v_i and v_s can be brought into good agreement with the values derived from curve-of-growth and line profile studies, which are of the order of 1.7 km/sec. However, although these observations by Adam may, after all, favour St. John's relativity-radial current hypothesis and support the view that the steady state of the solar atmosphere is maintained by a microscopic circulation associated with the solar granulation, they suffer from the disadvantage that they were obtained with low telescopic resolution, due to the necessity of admitting light into the

1) Schwarzschild, M., Rogerson, J.B., jr. and Evans, J.W.: Astron. J. 63, 313, 1958.

spectrograph from an area of 3 mm diameter on the 180 mm diameter Oxford solar image in order to limit the exposure time required to within reasonable limits[*]. In consequence, the disk positions nearest to the edge of the disk are subject to very large uncertainties.

This deficiency has been compensated by a very recent determination by Adam[1] of the limb effect of three FeI lines close to 6300A made with the new Oxford 35 metre telescope and high-dispersion grating spectrograph, which yielded spectra of very well specified disk positions with an exposure time of only 40 seconds, and permitted the extension of accurate limb effect measurements right up to the extreme limb of the Sun. The mean wavelength difference between this latter position and the centre of the disk corresponds to a velocity shift of 0.53 km/sec.; when this is combined with a centre-arc displacement of 0.31 km/sec., which Adam considers to be a likely value, there results an absolute shift of 0.84 km/sec. - about 0.20 km/sec. in excess of the relativity value. Thus the most recent and most reliable observational data confirm and strengthen Evershed's discovery of a real limb excess which - as Adam remarks - "is quite inexplicable on the radial current theory."

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1) Adam, M.G.; M.N., 119, 460, 1959.
*) The exposure times varied between 2^m and 8^m for the plates measured.

5. Schröter's Two-Stream Model and its Application to the Solar Red Shifts.

A sophisticated investigation of the relativity-radial current hypothesis has recently been carried out in Potsdam by Schröter[1], who took into account the various stratifications in velocity and temperature, the dependency of the velocities on depth, and the physics of the line production, in his attempt to explain the observed shifts. Schröter considered the introduction of such additional factors to be very essential, and necessitated by the observed dependency of the Sun-arc shifts on the line width $\frac{W_\lambda}{\lambda}$ and by the deviation of the limb effect from a cosine law. With regard to these observed features he writes:- "These two long-known facts are in contradiction to the demands of the theory of relativity, so that one must either negate the existence of the relativistic red shift, or one must try to explain the differences between observations and theory as due to the influence of additional solar physical effects. We shall adopt the latter point of view..."

On the basis of these considerations, Schröter built up his Two-Stream Model of the solar atmosphere by determining the radial convection streaming which would result

1) Schröter, E.; Zeits.f.Ap., 41, 141, 1957.

from temperature variations deduced from observations of the granulation: this procedure involves the adoption of values for the size and intensity ratio of the granules, and of the absolute measurements of the change in wavelength with position along a radius of the solar disk. Schröter supposes that:-

1) Each Fraunhofer line is composed of two original lines; one line contour being derived from the granula, and the other from the intergranula.
2) Both contours exhibit the Einstein effect; and so the mixed contour will also contain the predicted displacement.
3) The observed equivalent width W_λ can be represented by an equation of the form:-

$$W_\lambda = \frac{H_\theta}{H_\theta + 1} W_\lambda{}^{g} + \frac{1}{H_\theta + 1} W_\lambda{}^{ig}, \text{ where } H_\theta = \frac{I_o{}^{g}(\theta)}{I_o^{ig}(\theta)}$$

and $I_o(\theta)$ denotes the continuum intensity at angle θ to the outward radial direction, defined for the optical depth τ_o at a wavelength of 5010A: $W_\lambda{}^{g}$, $W_\lambda{}^{ig}$ are the equivalent widths produced in the granulum and intergranulum respectively, which differ because of the different physical conditions prevailing in these regions. The weights with which the granulum shift enters in the mean value will also vary; moreover, they depend on the parameter θ and contain the centre-limb variation of the continuum in the granulum and intergranulum.

In order to reduce the arbitrariness resulting from the presence of so many variable factors in his theory, Schröter first tested whether the above assumptions were compatible with certain independent observational data before applying them to the problem of the red shifts. He found that his Two-Stream Model yielded a satisfactory representation of the centre-limb variation of the average unresolved continuum (i.e. limb darkening), and provided an excellent representation of the centre-limb variation of the intensity in the wings of strong lines, as well as accounting for that of the equivalent width of weak lines, which had previously raised difficulties in model description that could never be well explained. The success of his model in describing the centre-limb variation of various types of lines which no homogene model of the photosphere in local thermodymamic equilibrium is capable of doing, and which Böhm's original Three-Stream Model[1] had done but less convincingly - satisfied Schröter that it could be justifiably employed for the purpose of explaining the observations of the solar red shifts.

According to his model, and in agreement with certain

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1) Böhm, K.H.; Zeits.f.Ap., 35, 179, 1954.

observations, the granulum is brighter than the intergranulum (in the continuum) by a factor of about 1.45; from which it can be inferred that if the granulum contour is displaced towards the violet, and that of the intergranulum towards the red, the net result will generally be to produce a violet shift. The resulting contour will be unsymmetrical, the form and amount of asymmetry (which determines the resultant shift) being a function of line intensity: the asymmetry is more effective as the relative contours become steeper. Hence strong lines may show a red or a violet shift, depending on whether the central intensity or the wings of a Fraunhofer line is chosen for measurement.

The limb effect results from the condition that the stratification in the granula is not the same as that in the intergranula, thereby producing a difference between the limb darkening of the two continuum levels, and in the centre-limb variation of the relative line contours. Schröter has demonstrated that this dependency, combined with the cosine law of projection and the constant relativistic shift, is capable of yielding a very good representation of Adam's 1948 observations; and that the Potsdam measures can also be quite well represented if, in accordance with his theoretical considerations, he assumes for the lines to which they refer a mean centre-arc shift of about 0.22 km/sec.

A consequence of Schröter's explanation is that every line with a different $\frac{W_\lambda}{\lambda}$ will give a different limb effect; over-relativistic shifts at the disk centre can be interpreted on the above basis, but at the Sun's limb all values must converge to the predicted gravitational shift, since there the physical effects should disappear. Thus this theory offers no explanation of the well-established excess shift at the limb. Additional evidence against Schröter's interpretation is contained in the values of the very accurate centre-arc shifts obtained at Oxford using the interferometric method of circular channels. From an examination of the relativity-radial current hypothesis on the basis of these observed data, Miss Adam[1)] has found that the resultant outward streaming velocity at the centre of the disk amounts to 0.184 km/sec. and 0.107 km/sec. for the lines contained in the Oxford (1948) and Potsdam investigations respectively, as compared with the corresponding value of 0.42 km/sec. adopted by Schröter for both cases. When substituted in Schröter's theoretical expression for the limb effect, these revised velocities no longer agree with the observations[2)]. Moreover, Adam has drawn attention to the fact that, contrary to expectation, her observations yield no correlation between

1) Adam, M.G.; M.N., 118, 106, 1958.
2) Adam, M.G.,;M.N., 118, 106, 1958, Fig.2.

the shifts and excitation potential. She has also examined the wavelength dependency of the velocity shifts using the A.O.-B.S. wavelength measures, and found that it bears no similarity to the required direct linear relation; but since we have reasons for believing that these values are strongly affected by observational errors, whereas we have also shown (Ch.II, 1) that St. John's data satisfy the above requirement, we do not regard this as a significant objection against accepting the relativity-radial current interpretation, as Adam has contended. Yet so long as one attempts to represent the observations according to the relativity-radial current hypothesis, one can never account for the limb excess: it would appear that in order to do so, some other cause must be invoked.

One such possibility may exist in the collisional displacements of solar wavelengths arising primarily from hydrogen-atom collisions, proposed by Spitzer[1]. Discussing the interpretation of the solar red shifts, Spitzer came to the tentative conclusion that "the measured shifts are simply the sum of the relativity red-shift and a collisional violet shift"; however, as a result of further researches carried out in Oxford during recent years[2] we know that one cannot

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1) Spitzer, L.: M.N., 110, 216, 1950.
2) Hart, A.B.; M.N., 114, 17, 1955.
Plaskett, H.H.; M.N., 119, 197, 1959.

ignore the contribution of organised Doppler motions at the photospheric level, as Spitzer originally believed. Since the laboratory lines as well as the solar lines are liable to suffer collisional shifts, the limb excess can be accounted for on the basis of Spitzer's interpretation as a consequence of a violet shift in the arc wavelengths of medium-intense and strong lines; nevertheless, in view of the theoretical and observational difficulties connected with the testing of this interpretation, we still cannot be sure of its validity.

6. Freundlich's Hypothesis.

An alternative proposal of quite a different character was put forward several years ago by Freundlich[1]. From a study of radial velocity measurements of certain B- and O-type stars, Freundlich formed the opinion that the wave-length shifts observed in stellar spectra are proportional to temperature (T). By comparing these values with the solar red shifts, he thought it likely that the effect varied as T^4: he attributed the shifts found from spectra of Giant stars to the much greater geometrical depth of the stellar atmosphere through which the light had passed before reaching a terrestrial observer. The same observational

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1) Finlay-Freundlich, E.; Phil.Mag., 45, 303, 1954.

features had been noticed by St. John[1], who interpreted the correlation of the shifts with temperature as due to the increase in the strength of the postulated radial currents, and considered that the low density (not the depth of atmosphere) was the major factor responsible for the shifts observed in Giant stars. St. John had therefore believed that these stellar observations provided additional support for his relativity-radial current hypothesis. Freundlich, on the other hand, realizing the difficulties confronting this interpretation, sought for another explanation and made the bold, though tentative, suggestion that these observations may provide evidence for a photon-photon interaction in the intense radiation field surrounding a star. Accordingly, he took T to be the temperature of the radiation field in which a photon of frequency ν travelling through an effective geometrical depth ℓ undergoes a change in frequency $\Delta\nu$ due to the numerous interactions with other photons "en route", and proposed the formula:

$$\frac{\Delta\nu}{\nu} = - A\, T^4\, \ell$$

to describe this phenomenon; A being a constant or proportionality whose physical nature was obscure.

The validity of Freundlich's formula was immediately

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1) St. John, C.E.; Ap.J., 67, 195, 1928.

questioned by McCrea[1], the Burbidges[2], Helfer[3], and shortly afterwards by Schröter[4], all of whom criticise the weakness of the observational evidence on which it is based. Yet despite some of these objections, ter Haar[5] defended it on principle, emphasizing that there are definite line shifts of physical origin in stellar spectra which cannot be satisfactorily accounted for by existing astrophysical theory. ter Haar was not prepared to accept Freundlich's interpretation, however, since he found that the cross-section of the postulated photon-photon interaction was much too small to be the mechanism responsible for the production of the shifts[6]. This view is also held by Melvin[7], who has shown that if a photon-photon interaction were responsible, the contribution of the radiation field well outside the Sun's atmosphere would be about a thousand times greater than that originating from within, a fact which is directly opposed to the observed steep increase of the limb effect of weak and medium-strong lines close to the edge of the disk.

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1) McCrea, W.H.; Phil.Mag. 45, 1010, 1954.
2) Burbidge, G. and M.; Phil.Mag. 45, 1019, 1954.
3) Helfer, H.L.; Phys.Rev. 96, 224, 1954.
4) Schröter, E.H.; Die Sterne, 32, 140, 1956.
5) ter Haar, D.; Phil.Mag. 45, 1023, 1954.
6) " " 45, 320, 1954.
7) Melvin, M.A.; Phys. Rev., 98, 884, 1955.

In his analysis of Adam's 1948 observations, Freundlich derived from a least-squares solution that:

$$\Delta\lambda(\theta) = 2.72 + 1.85 \sec \theta$$

where the mean absolute wavelength shift $\Delta\lambda(\theta)$ and the constants of this two-term representation are expressed in milli-Angstrom units (mA); but he was unable to offer a plausible explanation of the first term, which contains the effects arising in the comparison between solar and laboratory wavelengths. Although this figure is only about $\frac{1}{5}$ of the Einstein effect at the wavelength region to which the observational data refer (6100A), it is nevertheless larger than the "interaction constant" involved in the second term with whose interpretation Freundlich's paper (1954) is concerned. The latter difficulty, however, has now been removed by a recent re-examination by the writer in collaboration with Professor Freundlich of the empirical evidence available from solar observations which led to the realization that this hitherto unexplained term is not an independent effect but constitutes a complementary part of the hypothetical effect itself, and arises from the fact that the solar atmosphere is in a state of radiative equilibrium.[1,2,3]

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1) Finlay-Freundlich, E.; and Forbes, E.G.; Ann.d'Ap., 19, 183, 1956
2) " " " " 19, 215, 1956
3) " " " " 22, 727, 1959

The major result of this joint research was to provide confirmation that the hypothetical formula can account completely for the available data on the solar red shifts, quite independently of the results deduced from stellar spectra, when written in the equivalent form:

$$\frac{\Delta\lambda}{\lambda} = C\, I_\lambda\, \ell$$

in which the Sun-arc wavelength shift $\Delta\lambda$, the continuum intensity I_λ and the length of path ℓ all refer to a given disk position $\theta (0 \leq \theta \leq \frac{\pi}{2})$ and C is a constant of proportionality analogous to the co-efficient A in the original formula. In order to emphasize the accuracy of the amended empirical representation, it is necessary to point out that Miss Adam's objection that the path-length hypothesis predicts a limb effect of more than twice the observed value at $\theta = \frac{\pi}{2}$ [1] is removed by our introduction of the continuum intensity, for this factor contributes towards reducing the calculated value of $\Delta\lambda(\frac{\pi}{2})$ to a value which is compatible with observation. For the same reason, we now require to correct our own view[2] that Walter Adams might have incurred a systematic error of about 0.5 mm in his estimate of the limb position at which he made his extensive observations of the limb-centre differences (1910), since these data can

1) Adam, M.G.; M.N., 119, 473, 1959.
2) Finlay-Freundlich, E.; and Forbes, E.G.; Ann.d'Ap., 19, 215, 1956

now be brought into line with other independent results without the need for such an assumption. Thus from an empirical standpoint, Freundlich's formula in its amended form must be accepted as yielding a very good approximation to the most reliable of the presently existing red shift observations referring to weak and medium-intense Fraunhofer lines. Thus it is not unreasonable to suppose that similar differences must also be implicit in the wavelengths found from the less reliable measurements of stellar spectra, as Freundlich originally contended. Yet the hypothesis that these red shifts are produced wholly by some interaction mechanism between photons and unknown particles in the outermost regions of the solar (or stellar) atmosphere has not been generally accepted since it would appear to preclude the existence of the gravitational red shift.

We finally conclude that the present situation is little better than it was over thirty years ago, when St. John proposed his relativity-radial current interpretation. We are now aware that wavelength displacements of the order required to secure agreement with observation may be associated with the collisional broadening of spectral lines, but various theories of this nature have yet to be explored before the significance of such a possibility can be assessed. From the observational standpoint, more reliable and extensive data are required. As a contribution, the writer has collected spectrographic material with the help of the facilities available at the Arcetri Observatory, Italy and at the Göttingen Observatory, Germany, with a view to investigating the wavelength and multiplet dependencies of the centre-limb wavelength variation of moderately intense Fraunhofer lines in the visible and near infrared regions of the solar spectrum. No attempt has been made to extend the observations to very weak or very strong lines, since this would have involved special techniques better suited to higher resolution equipment. The rest of this thesis will be concerned with describing this research and interpreting the results derived from it.

CHAPTER III.

An Observational Investigation of the Centre-limb Wavelength Variation of Moderately Intense Fraunhofer Lines in the Visible and Near Infrared Solar Spectrum.

At the outset of this investigation, considerable attention was given to the problem of developing a light-source capable of producing sharply defined wavelengths unaffected by systematic observational errors, which would serve as absolute standards for the calibration of the centre-limb wavelength variation of the corresponding solar absorption lines. It is now generally recognised that the Pfund arc (cf. Ch.I.2), although still useful as a comparison source for the approximate determination of stellar wavelengths, is unsuitable for solar work, in which an absolute accuracy to the nearest ± 1 mA is often demanded: the main disadvantages being that the high operating temperature (near 6,300°K) causes the spectral lines to appear diffuse, thus making visual micrometer measurement a matter of some difficulty, and that the existence of a voltage gradient between the iron electrodes at atmospheric pressure might produce appreciable asymmetries in the profiles of lines influenced by the Stark effect. (e.g. self-reversed lines involving low energy levels and diffuse lines involving high energy levels.) These objections can, however, be overcome by adopting the type of light source described by Meggers and Westfall[1] and subsequently developed at the National Bureau of Standards (NBS) in Washington D.C., U.S.A.[2] which consists of an electrodeless quartz tube containing a few milligrams of iron (or other metal) halide and an inert carrier gas at 2-5 mm. Hg pressure. When such a tube is placed inside the waveguide of a CW magnetron and excited by microwaves, iron lines are emitted under coditions of low pressure and moderate temperature and consequently the internal accuracy of measurement can be increased by a factor of 2 to 3; moreover, the possibility of wavelength displacements due to the Stark effect is eliminated since there is no voltage gradient along the tube.

With the help of firms in Florence and Göttingen which specialise in making lamps for advertising and scientific purposes respectively, two batches of electrodeless tubes containing iron iodide and neon or argon gas were prepared

1) Meggers,W.F. and Westfall,F.O.; J.Research NBS,44,447,1950.
2) Stanley,R.W. and Meggers,W.F.; J.Research NBS, 58, 41,1957.

in accordance with the instructions given by Corliss, Bozman and Westfall[1], using the recommended type of exciter unit: viz. the Raytheon Manufacturing Company's Microtherm, which contains a QK59-62 magnetron operating at 2450 Mc with 100 watt maximum power output. Yet despite all efforts to coax the tubes to function, no iron spectrum was observed either visually or photographically in the first order of the Arcetri spectrograph. It was originally thought that the reason for this failure might be associated with the inefficiency of the roof-shaped waveguide supplied with the generator*) in concentrating the microwave energy inside the lamps, and so this was replaced by a carefully-matched hollow tubular waveguide designed by Sig. M. Piattelli and constructed by an electronic engineering firm in Florence; unfortunately, however, no better result was obtained. The negative outcome of these experiments meant that in order to utilise to the best advantage the time allotted to the investigation, we were left with the alternative of comparing the wavelengths of iron lines at various positions on the solar disk with those of the Pfund arc, or with the corresponding solar wavelengths at the disk centre. In view of the doubtful validity of the former procedure, it

1) Corliss, C.H., Bozman, W.R. and Westfall, F.O.: J.O.S.A., 43, 398, 1953.

*) This instrument was really intended for medical diathermy.

was decided to concentrate entirely upon a programme of observations of relative wavelength differences only. In this chapter, we shall be concerned with describing the instrumentation and method of spectral comparison which have been employed for this purpose.

A. THE ARCETRI OBSERVATIONS.

1. The Arcetri Solar Tower.

No major alteration has been made to the Arcetri solar tower since its construction was completed in 1925; full information and diagrams concerning the refracting telescope and the spectrographic equipment have been published by G. Abetti,[1,2] so it should suffice to mention only those details directly relevant to the present discussion. The tower consists of a re-inforced concrete structure surmounted by a 4 metre diameter capola, which supports two plane mirrors and the objective: the whole construction rises to a height of 25 metres above ground level, and has proved itself to be very stable against the action of the wind. The coelostat and secondary mirrors are both made of Crown glass, and each is 42 cm. in diameter: they are aligned in the plane of the meridian, the former being mounted in such a way that it can be raised or lowered (according to the change of season) in a plane inclined to the horizontal by the angle of latitude of Arcetri (i.e. $43^{\circ}\ 45'$). The secondary mirror, which is mounted on a

1) Abetti, G.; Pubbl.Oss.di Arcetri, 43, 13, 1926.
2) Abetti, G.: Il Sole(2nd Ed.), Plates6-9 and Figs.10, 11, 1951(Faber and Faber)

concrete column, reflected the bundle of rays from the Sun vertically downwards on to the 30 cm. aperture objective of 18 metre focus normally employed when high telescopic resolution is desired. The type of mounting described above suffers from the disadvantage that from about the middle of August until the middle of April the coelostat is occulted around mid-day either by the column of the secondary mirror or by the secondary mirror itself, depending upon the Sun's altitude; however, this is unimportant since the best hours for observing are generally from 7-9 a.m. and from 4-6 p.m., when the temperature gradient between the upper and lower atmospheric layers is small.

The coelostat mirror is driven mechanically by a weight suspended from a cable wound round a metal drum geared to the axis on which the mirror is mounted. Its rate of rotation is one-half that of the Earth's axial motion (with respect to the Sun), in order that the solar image formed after reflection from the coelostat and secondary mirrors should remain stationary with respect to the spectrograph slit. This rotary movement is controlled by a governor, but there is in addition a differential gear which, when activated from the base of the tower by means of two relays and a small electric motor, accelerates or retards the motion of the coelostat mirror and thereby causes the image to be displaced in the East-West direction. A similar arrangement coupled to the column supporting the secondary mirror controls its position in the North-South direction. The image can be kept in focus by means of manually-operated electrical controls which drive the circular metal carrier supporting the objective upwards or downwards (as required) along two vertical steel columns.

The two most important factors limiting the telescopic resolution are:-

i) The heating effect of the Sun's rays in altering the effective focal length, and hence the image size.

ii) The scattering of light by dust particles etc. in the Earth's atmosphere.

It was estimated that the latter effect produces an uncertainty in defining the edge of the Sun's image of about $\pm\frac{1}{4}$ mm under normal observing conditions; but this type of error is inherent in all observations of this kind, and cannot be entirely avoided so long as one is using terrestrially-based equipment. It would appear, however, that the heating of the lens imposes a more serious limitation on the accuracy attainable than does the atmospheric scintillation; for direct measurements on the solar image made at times of exceptionally good "seeing" showed a systematic daily variation of about ½mm. in the value of its radius, ranging between a minimum of 83.3 mm. in the early morning or late afternoon to a maximum of 83.8 mm. around mid-day. These figures imply a corresponding change in the focal length of the objective of about 10 cm., from 1790 cm. to 1800 cm., in satisfactory agreement with the value of 1785 cm. found for the focal length of the lens when it was initially tested under laboratory conditions using an artificial light-source. Thus it was realised that - short of replacing the Crown glass objective by another made of quartz, which is insensitive to temperature changes - an absolute precision of under ±½ mm. in the definition of a given disk position is scarcely to be expected.

2. Auxiliary Instruments.

(a) The Guiding Device.

A device with which a consistent setting on the edge of the Sun's image could be obtained was designed and constructed by the writer: it is composed essentially of a long steel plate of 10 cm. width and ½ mm. thickness; a thin sheet of brass 26 cm.× 11 cm. from which a semi-circular portion of 165 cm. diameter (slightly less than the minimum diameter of the solar image formed by the 18 metre focus objective) had been cut; and a 4 cm. wide brass strip attached to the brass sheet by screws passing through a 10 cm. long slot cut in the steel plate. The latter was bolted to a strong iron frame whose position with respect to the spectrograph slit remained fixed, hence the other two components could be set at various positions by sliding them backwards and forwards along the length of the plate. A paper scale stuck on to the brass stfip enabled the exact amounts of such displacements to be known to within $\pm\frac{1}{2}$ mm., and reference scratches engraved around the edge of the semi-circular aperture allowed the observer to guide upon the edge of the image to within the same limits of accuracy. A schematic drawing of the instrument is given in Fig.4a (the smaller slot being cut in the plate merely in order that light from the selected portion of the Sun's disk would reach the spectrograph), and its arrangement with respect to the slit is illustrated in Fig.4b. A small systematic discrepancy in positioning the image above the reference grooves on the guiding plate occurs due to the fact that the Sun's apparent angular diameter varies from a maximum at perigee (about December 22nd) to a minimum at apogee (about June 22nd); but the consequent seasonal changes in the image diameter are always less than the daily variations occasioned by the heating effect on the telescope objective, hence it was

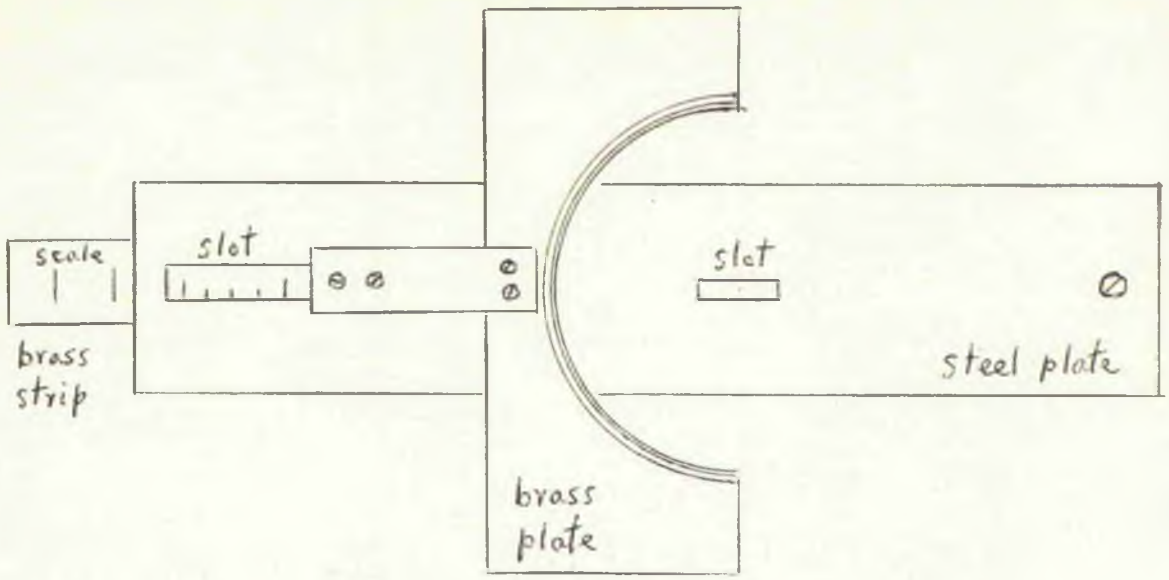


FIG.4a: The Guiding Instrument (Plan view).

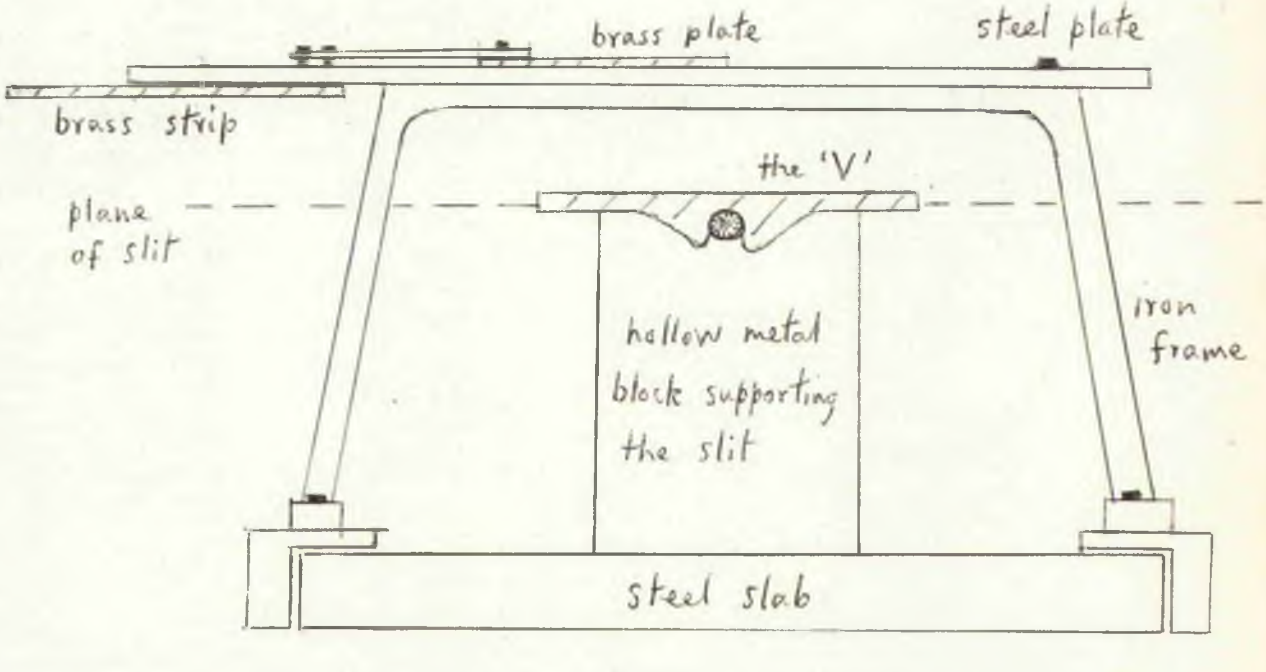


FIG.4b: The Arrangement of the Guiding Instrument with respect to the Spectrograph Slit.

considered unnecessary to make quantitative allowances for the former source of uncertainty.

(b) The "V".

This nomenclature has been coined to describe an accessory instrument available at the Arcetri Observatory whose characteristic feature is a V-shaped wedge which fits neatly into a groove of the same shape cut in a brass plate. The wedge and plate are both mounted upon a metal block of 180 mm. diameter circular section. This block is surmounted by a thin rim in which grooves are cut which serve as reference rings above which the image - after first being brought into focus by observing the definition of features such as small sun-spots, etc. - can be set so that its centre co-incides with the middle of the 75 mm. long slit: it is clamped firmly by two screws, one on either side of the hollow tower supporting the slit (cf. Fig.4b). The wedge can be moved laterally through specified distances alongside a scale (graduated in half-millimetres) simply by turning a screw. When the screw is turned in a clockwise sense, the wedge pushes the plate in front of it; but when the screw is turned anti-clockwise, the wedge moves back leaving the plate where it was, and at the same time uncovering a portion of the slit. By this means, either the central portion of

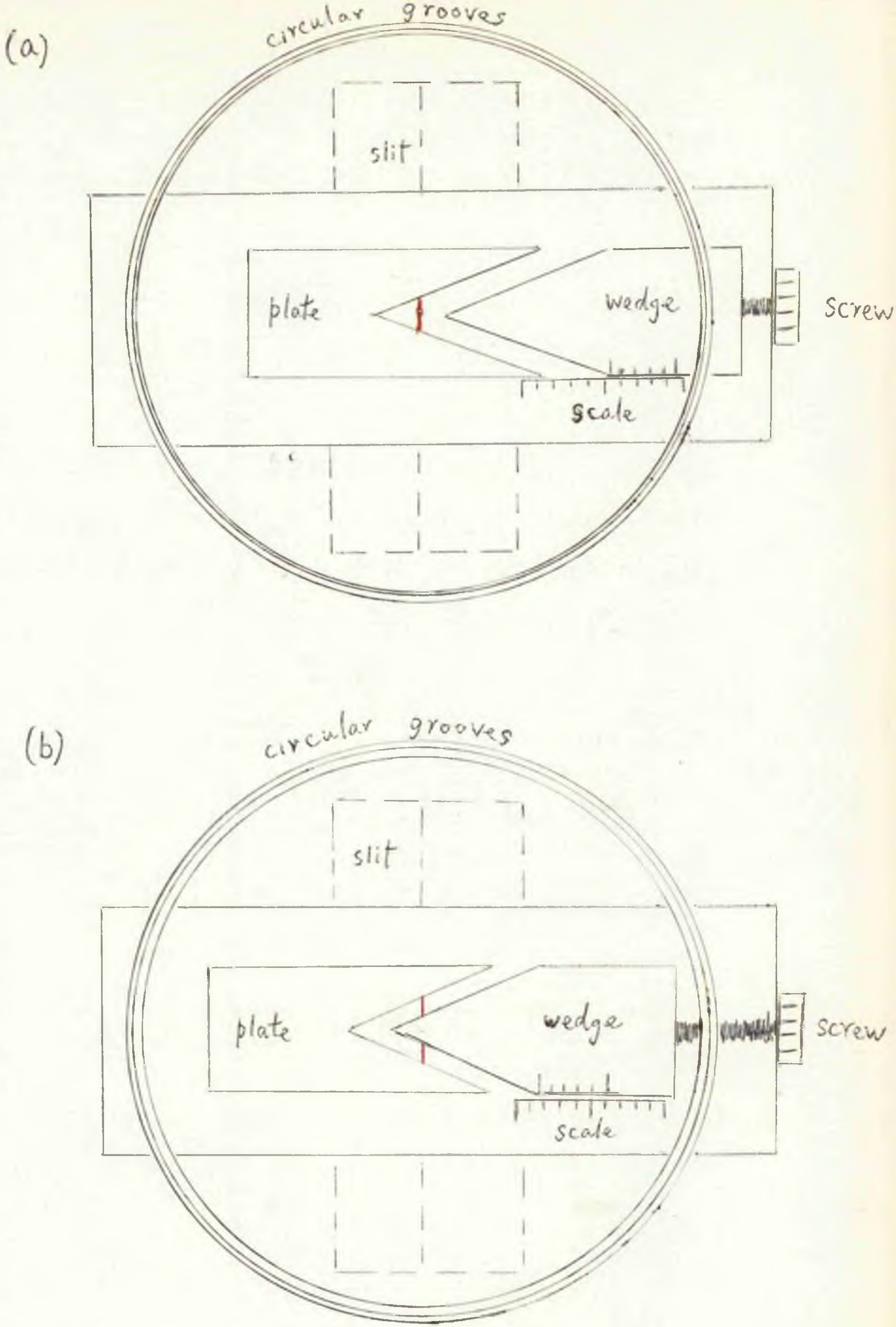


FIG.5: Method of Spectral Comparison using the "V" (Plan vie

the slit or the parts adjacent to it can be exposed, as indicated in Figs.5 a, b respectively.

3. The Arcetri Spectrograph.

The spectrographic equipment installed at the Arcetri solar tower was designed to serve also for spectroheliographic work, hence it can easily be orientated around a vertical axis into any desired position angle whose value can be set on a graduated circle. After passing through the horizontally-mounted slit (cf. Fig.4b), which is adjustable in width to the nearest micron, light from a selected portion of the Sun's image eneters a blackened pit where it is collimated by a Crown glass lens of 15 cm. aperture and 4 metre focal length; the parallel beam is then reflected off a plane mirror (25 cm. × 16 cm.) situated vertically below the lens on to the 12 cm. × 11 cm. ruled surface of a Babcock grating with 600 rulings/mm. The angle of this grating can be varied mechanically so that the light dispersed at some given wavelength in one of the lower spectral orders could be made to pass through a second lens similar to the first and thus be brought to a focus in the plane of the slit. The resulting spectra may either be viewed visually with the aid of an eyepiece, or photographed in a conventional manner. The spectrographic arrangement is illustrated schematically in Fig.6a.

The optical components are supported by a rigid metal framework built round four vertical steel columns suspended (inside the pit) from the steel slab to which the metal block containing the slit is attached (cf. Fig.4b). The two lenses were made by Zeiss. Optical tests made jointly by Professors Abetti and Ronchi have shown that their chromatic aberrations are small, the relative differences in focal length from 3500A-5000A and from 5000A-6500A being only 4-5 mm. The results obtained from focus plates taken by the writer in the 3rd order spectrum of the Arcetri spectrograph confirm that this is true in the case of the camera lens: hence we should be safe in asserting that the largest colour aberration of either lens within the visible wavelength range must amount to little more than one-thousandth part of its focal length. A comparison of focus plates taken in March and April 1958 with others collected in July and August of the same year indicates a slight seasonal variation in the focal length of the camera lens which might be interpreted as a tendency for this quantity to increase with increasing air temperature inside the pit; however, the effect is small (about 2 mm.) and should not affect values of relative wavelength shifts found from plates taken at different time of year. Similarly, it may be inferred that the daily temperature variations of the air inside the pit are also too small to produce systematic errors in the observational data, particularly since the volume of air in the pit is large enough to preclude the possibility of a sudden temperature rise*. As an additional precaution, however, the trap-door through which access is gained to the pit was closed while the observations were in progress, but left open during the night to allow the air to circulate freely around the apparatus.

4. Conditions for obtaining Maximum Dispersion and Resolving Power with the Arcetri Spectrograph.

Let us suppose that a beam of light entering the spectrograph impinges upon the grating at an angle of incidence i. Let φ be the angle of diffraction of the light at wavelength λA, b the spacing between two consecutive rulings on the grating surface, and k the spectral order; then straightforward geometrical considerations yield the well-known grating formula

$$b(\sin i - \sin \varphi) = k\lambda \qquad (1)$$

* When the solar tower was constructed it was originally planned to instal a 9-metre focus Littrow-type (i.e. autocollimating) spectrograph in the pit, consequently the latter was dug to a depth of 9.4 metres, more than twice that required to house the present spectrograph.

Defining $m = \frac{1}{b} = 600$ mm.$^{-1}$, equation (1) may be otherwise written as

$$\sin i - \sin \varphi = mk\lambda \qquad (2)$$

For a fixed angle of incidence (i = constant), differentiation of (2) gives the angular dispersion

$$\left|\frac{d\varphi}{d\lambda}\right| = \frac{mk}{\cos\varphi} \qquad (k = 1,2,3,\text{etc.}) \qquad (3)$$

Imposing the minimal conditions $\frac{d}{di}(\cos\varphi) = 0$ and $\frac{d^2}{di^2}(\cos\varphi) > 0$ upon

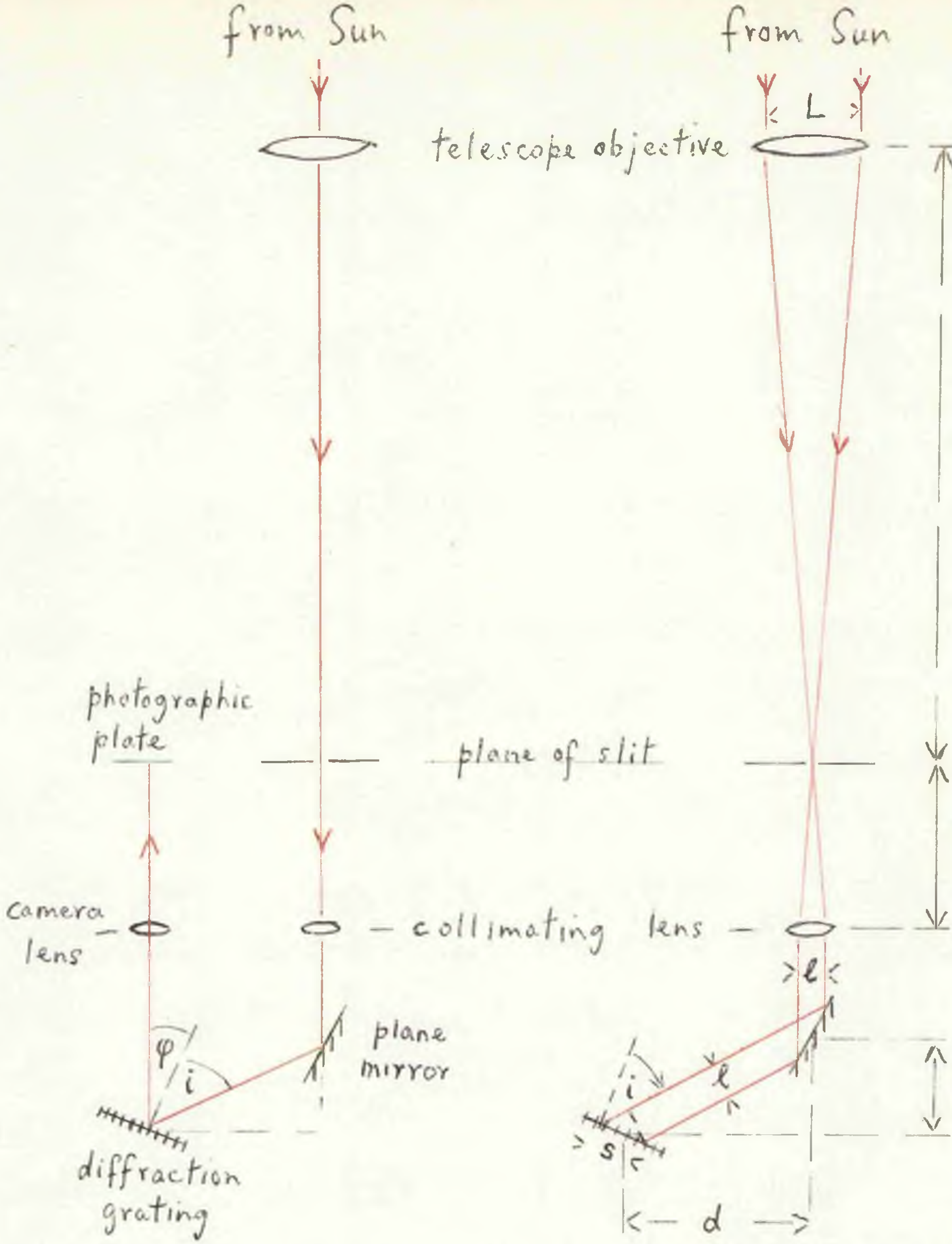
$$\cos \varphi = \sqrt{1 - (\sin i - mk\lambda)^2} \qquad \text{(using 2)}$$

one finds that within the physically permissible range $0 \leq i \leq 90^{\circ}$, $\cos \varphi$ has a minimum value only when $i = 90^{\circ}$. It follows from (3) that the angular dispersion attains a relative maximum at grazing incidence, and that its absolute value increases with spectral order.

Maximum spectrographic resolution occurs when the parallel beam of light reflected off the mirror strikes the grating at an angle i_o such that

$$\cos i_o = \frac{\ell}{s} \qquad (4)$$

where s denotes the ruled length of the grating, and ℓ the diameter of the beam. From an inspection of Fig.6b one realises that when $i < i_o$ the resolving power becomes smaller since not all of the ruled surface is illuminated ; and when



(a) **Path of Single Ray along the Optical Axis.**

(b) **Path of Two Limiting Rays at the Edge of the Beam.**

FIG.6: Schematic Representation of the Arcetri Solar Tower and Spectrograph (not drawn to scale).

$i > i_o$, the resolving power remains the same although the spectral intensity is weakened due to the incident radiation flux being spread over an area larger than that of the ruled surface. The effective value of ℓ is given by the condition (cf.Fig.6)

$$\ell = \frac{L}{F} \times f \tag{5}$$

in which L, F respectively represent the aperture and focal length of the telescope objective, and f denotes the focal length of the collimating lens.

Combining (4) and (5), one obtains,

$$\cos i_o = \frac{L}{F} \times \frac{f}{s} \tag{6}$$

The numerical values of the quantities contained in the right-hand side of (6) have already been quoted in the previous discussion ; expressed in the same units (centimetres) they are -

$L = 30$ cm. $\qquad f = 400$ cm.

$F = 1800$ cm. $\qquad s = 12$ cm.

From equation (6) one finds that $\cos i_o = \frac{5}{9}$, or $i_o = 56^o$. Thus to obtain maximum spectrographic resolution together with maximum light-gathering power, a grating angle of incidence of about 56^o is required.

Experience during several years in making spectroheliographic observations with the Arcetri solar tower and spectrograph led to the mirror being secured at a height $h = 19$ cm. above the grating, the horizontal separation $d = 27$ cm. being fixed by the design of the spectrograph. We now require to examine the conditions under which this configuration may be deemed suitable for spectrographic work in which high resolution and dispersion are essential requirements.

From an inspection of Fig.6a it can be seen that

$$\tan(90 - \overline{\varphi + i}) = \frac{h}{d}$$

$$\text{or} \qquad \tan(\varphi + i) = \frac{d}{h} = \frac{27}{19} = 1.42$$

This relation yields $\varphi + i = 55^{\circ}$. ...(7). In particular, for $i = i_{o} = 56^{\circ}$, $\varphi_{o} = -1^{\circ}$. i.e. the plane of the grating is practically horizontal. In general, substituting (7) into the grating formula (2),

$$\sin i - \sin(55 - i) = mk\lambda.$$

Hence $\qquad 1.57 \sin i - 0.82 \cos i = 0.6 \times 10^{-4} k\lambda \quad (8)$

where λ is expressed in angstrom units. Using this equation, one can find the values of i ($0 \leq i \leq 90^{\circ}$) which correspond to any given wavelength and spectral order.

The results of calculations for the first four orders of the Babcock grating are illustrated in Fig.7. Now maximum resolving power is attained only when $i \geq 56^{\circ}$, and

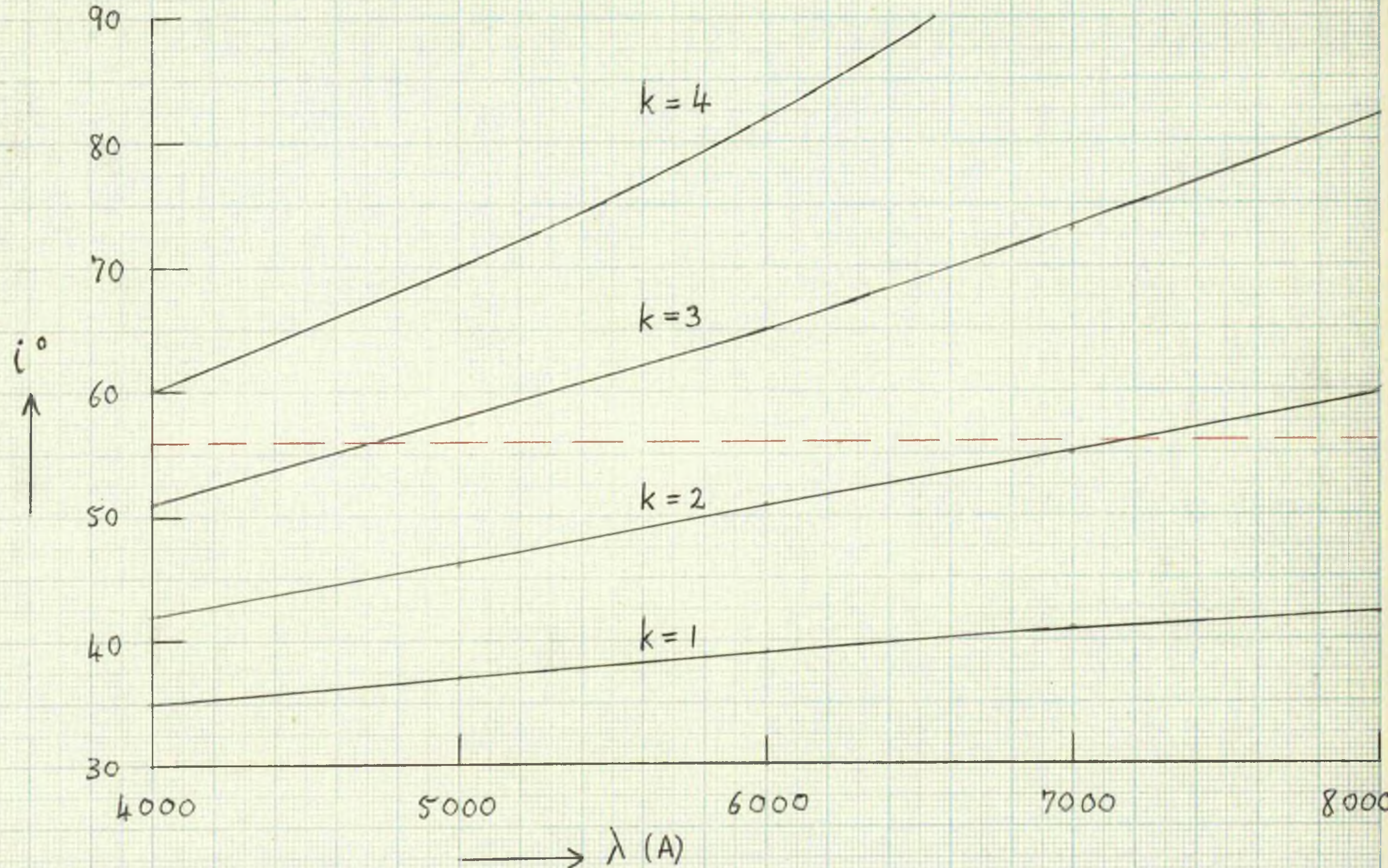


FIG.7: Relationship between Grating Angle of Incidence and Wavelength for different Spectral Orders

the graphs show that this condition is never satisfied for visible wavelengths in the 1st and 2nd orders, however, when $i > 70^{0}$ the spectral intensity diminishes so rapidly that a wide slit and long exposure times are necessary for photographing the spectra, and this limitation renders the 4th order unsuitable for observations at wavelengths greater than 5000A. For the same reason, the 2nd order is to be preferred for work relating to the near infrared region, since it yields maximum resolution combined with a high spectral intensity. But this latter criterion is not the sole one that has to be considered; the linear dispersion must also be as high as possible in work demanding such high precision. Bearing in mind this second requirement, it was decided to use the Göttingen solar tower and telescope for collecting observational material in the near infrared, while the 3rd order of the Arcetri grating was deemed the most suitable for the purpose of investigating the centre-limb wavelength variation within the limits of the visible spectrum.

5. Description of the Limb Effect Observations in the Visible Wavelength Range.

The linear dispersion in the 3rd order of the Arcetri spectrograph was found to vary but little: viz. from 1.48 A/mm. in the blue to 1.46 A/mm. in the red region of the spectrum. By definition, the theoretical resolving power in this order is

$$\frac{\lambda}{d\lambda} = mks = 600 \times 3 \times 120 = 216{,}000.$$

In view of the moderate dispersion and resolution of the Arcetri equipment, as evidenced by the above figures, a tangential slit was preferred to a radial one in this programme of observations becuase it was felt that the former procedure is the less likely to involve difficulties in the measurement and reduction of the photographic material.

During the summers of 1958 and 1959 over 100 plates, centred near wavelengths of 4400A, 5300A, 6100A, and 6300A were collected with the instrumentation described in the previous part of this chapter. Each plate contained spectral comparisons between the centre of the Sun's disk and 9 selected points along either the North or South polar radius; the relationship between disk position and various functions of the angle of emergence (θ) of the solar radiation being given in Table 5. With the help of the "V" (Fig.5), the limb spectra were always photographed between two (simultaneously exposed) spectra taken from the central portion of the Sun's image; and since the magnification of the spectrograph is approximately unity, each spectrum was the same width as the selected lengths of slit: viz. 1½ mm.

TABLE 5.

Relationship between Disk Position and Functions of the Angle of Emergence θ

Disk Position	1	2	3	4	5	6	7	8	9
sin θ	0.24	0.42	0.60	0.72	0.84	0.90	0.96	0.98	0.99
cos θ	0.97	0.91	0.80	0.69	0.54	0.43	0.27	0.22	0.155
log cos θ+10	9.99	9.96	9.90	9.84	9.73	9.63	9.43	9.34	9.19
sec θ	1.03	1.10	1.25	1.45	1.86	2.33	3.76	4.59	6.47

In order to save time between consecutive comparisons, the centre spectra were photographed alternately before and after the corresponding limb spectrum, for in this way it was not always necessary to adjust the "V" and/or the position of the image after the completion of each comparison. It must be emphasised that neither the plate-carrier nor the spectrograph were disturbed in any way during the course of a given comparison, since this is a very important condition that must be fulfilled if high accuracy is to be attained.

The spectrograms were generally over-exposed in order to increase the contrast and thus enable consistent settings of the cross-wire to be made on the line core; this meant that they were unsuitable for photometric work, but a detailed study of the line profiles was also deemed inadvisable on the grounds of the limitations imposed by the spectrographic resolution and time at the writer's disposal. The exposure times adopted with a slit width of 0.030 mm. in the blue, green, orange and red regions of the visible spectrum varied from 1-3, 5-14, 1½-7, 1½-7 minutes respectively between the disk centre and the outermost limb position, the increase in time being required to compensate for the weakening of the continuum intensity resulting from the phenomenon of limb darkening. Suitable filters were placed in front of the slit so as to eliminate the possibility of interference from the 2nd and/or 4th order spectra.

From the entire observational material collected at Arcetri, four sets of 10 plates - five containing North limb comparisons, and five containing South limb comparisons along the polar diameter at each of the four wavelength regions concerned - were chosen for reduction; the particulars relating to these are given in Tables 6-9. In all 28 medium-intense solar FeI lines of varying levels of excitation, and 2 weak atmospheric lines contained on these plates were finally selected for measurement on the criteria of their symmetry and apparent freedom from blends as judged from a visual examination of their intensity profiles in the Utrecht Atlas. The FeI lines are classified according to wavelength and their observed laboratory wavelengths are listed in Table 10 along with their associated atomic properties and respective line strengths: the latter quantity was found from Allen's estimates of equivalent widths derived from the measured central intensities[1], while the other information is copied from Moore's Multiplet Table[2]. There is, however, no guarantee that a careful photometric study of the intensity profiles of some of these lines, made with very high spectrographic resolution, would not reveal the presence of asymmetries in the contours or blending with hitherto unresolved weak lines; consequently, the material ought to be discussed on a statistical basis, since the values obtained from the measurement of individual lines may be influenced by such effects.

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1) Allen, C.W.; Memoirs Comm.Solar Obs.**1**, (6),1934; **2**,(1),1938
2) Moore, C.; A Multiplet Table of Astrophysical Interest, rev.ed.,Contr.Princeton Univ.Obs., No.20, 1945.

TABLE 6.

Particulars of Plates centred near 4400A.

Plate Desig	Date	Continental Time (CT)	Limb Observed	Atmospheric Conditions
B1	5.7.59	9.45 - 10.40	North	Very clear
B2	5.7.59	10.40 - 11.30	"	"
B3	5.7.59	11.40 - 12.30	"	"
B4	5.7.59	14.25 - 15.25	"	"
B5	5.7.59	15.40 - 16.35	"	"
B6	9.7.59	14.20 - 15.05	South	Clear
B7	12.7.59	10.05 - 11.00	"	Hazy
B8	12.7.59	11.55 - 12.35	"	Slightly Hazy
B9	12.7.59	12.40 - 13.35	"	"
B10	19.7.59	12.45 - 13.30	"	Hazy

TABLE 7.

Particulars of Plates centred near 5300A.

Plate Desig	Date	Continental Time (CT)	Limb Observed	Atmospheric Conditions
G1	16.8.58	9.15 - 11.25	North	Windy, but clear
G2	16.8.58	13.45 - 15.45	South	"
G3	17.8.58	8.45 - 11.05	"	Clear
G4	17.8.58	13.25 - 15.30	North	"
G5	8.9.59	11.35 - 14.10	"	Windy, but clear
G6	9.9.59	(11.05 - 11.40 12.50 - 14.25)	South	Clear
G7	10.9.59	13.05 - 15.25	North	Hazy, but calm
G8	11.9.59	13.15 - 15.40	South	Very clear
G9	12.9.59	13.10 - 15.50	"	Hazy
G10	13.9.59	9.00 - 11.40	North	Clear

TABLE 8.

Particulars of Plates centred near 6100A.

Plate Desig	Date	Continental Time (CT)	Limb Observed	Atmospheric Conditions
01	5.8.58	13.30 - 14.45	North	Windy, but clear
02	10.8.58	8.55 - 9.55	South	Very clear
03	11.8.58	15.10 - 16.15	North	"
04	27.8.58	15.30 - 16.30	South	"
05	21.8.59	11.55 - 12.55	"	Slightly hazy
06	21.8.59	13.05 - 14.10	"	"
07	21.8.59	14.15 - 15.25	"	"
08	25.8.59	11.35 - 12.45	North	"
09	25.8.59	12.50 - 13.55	"	"
010	25.8.59	15.00 - 16.00	"	"

TABLE 9.

Particulars of Plates centred near 6300A

Plate Desig	Date	Continental Time (CT)	Limb Observed	Atmospheric Conditions
R1	27.7.58	16.20 - 18.00	North	Hazy
R2	28.7.58	7.45 - 9.15	"	"
R3	30.7.58	16.05 - 17.25	South	Windy, but clear
R4	10.8.58	13.25 - 14.25	"	Clear
R5	28.8.58	7.50 - 8.50	North	Very clear
R6	28.8.58	16.55 - 18.00	"	"
R7	14.9.58	16.05 - 17.10	South	Windy
R8	15.9.58	16.30 - 17.40	"	Very clear
R9	4.8.59	12.35 - 13.35	North	Hazy
R10	7.8.59	15.45 - 16.45	South	Clear

TABLE 10.

Description of FeI Lines measured on the Arcetri Plates.

λ(A)	$\frac{W}{\lambda}\times10^{6}$	Multiplet No.	Excitation Pot (χ) χ_{low}	Excitation Pot (χ) χ_{high}	J	Multiplet Designation
4389.244	18.2	2	0.05	2.86	3-2	$a^5D-z^7F^o$
4442.343	35.8	68	2.19	4.97	2-2	$a^5P-x^5D^o$
4447.722	34.2	68	2.21	4.99	1-1	"
4484.227	21.4	828	3.59	6.34	3-4	$z^5P^o-g^5D$
4485.679	18.1	830	3.67	6.42	1-1	$z^5P^o-e^5P$
4489.741	25.9	2	0.12	2.87	0-1	$a^5D-z^7F^o$
5247.052	13.0	1	0.09	2.44	2-3	$a^5D-z^7D^o$
5250.212	13.5	1	0.12	2.47	0-1	"
5322.049	11.1	112	2.27	4.59	2-3	$a^3P-y^3F^o$
5324.185	47.8	553	3.20	5.52	4-4	$z^5D^o-e^5D$
5364.874	23.1	1146	4.43	6.73	2-3	$z^5G^o-e^5H$
5365.403	14.9	786	3.56	5.86	5-4	$a^1H-z^1G^o$
5367.470	25.0	1146	4.40	6.70	3-4	$z^5G^o-e^5H$
5379.580	10.2	928	3.68	5.97	4-5	$b^1G-z^1H^o$
5383.374	27.3	1146	4.29	6.59	5-6	$z^5G^o-e^5H$
6055.987	11.6	1259	4.71	6.75	3-4	$y^3D^o-f^3F$
6065.487	21.8	207	2.60	4.63	2-2	$b^3F-y^3F^o$
6136.999	11.3	62	2.19	4.20	2-1	$a^5P-y^5D^o$
6151.624	8.0	62	2.17	4.17	3-2	"
6157.734	10.1	1015	4.06	6.06	4-4	$c^3F-w^3F^o$
6173.343	10.9	62	2.21	4.21	1-0	$a^5P-y^5D^o$
6246.334	19.2	816	3.59	5.56	3-3	$z^5P^o-e^5D$
6265.140	13.1	62	2.17	4.14	3-3	$a^5P-y^5D^o$
6297.800	11.0	62	2.21	4.17	1-2	"
6301.515	18.4	816	3.64	5.60	2-2	$z^5P^o-e^5D$
6302.507	12.5	816	3.67	5.63	1-0	"
6335.335	16.7	62	2.19	4.14	4-3	$a^5P-y^5D^o$
6358.692	11.9	13	0.86	2.80	5-6	$a^5F-z^7F^o$

B. THE GÖTTINGEN OBSERVATIONS.

1. The Göttingen Solar Tower and Spectrograph.

Full details concerning the tower telescope and the spectrographic equipment installed at Göttingen have been published quite recently by ten Bruggencate et al.[1,2] so only those which are directly relevant to the present investigation are repeated below:-

Light from the Sun is reflected by two 65 cm. diameter plane mirrors mounted on a rotary base, vertically downwards into a Cassegrain system comprising a parabolic primary mirror of 45 cm. aperture and 3 metre focus and a hyperbolic secondary of 10 cm. diameter. A short distance above the primary mirror (along the optical axis) is situated a 10 cm. Coudé mirror which deflects the beam in a horizontal direction on to the spectrograph slit. The effective focal length of the system is 24 m., which yields an image of 220 mm. diameter. By a suitable manipulation of electronic controls, light from any selected portion of this image can be made to pass through the (vertically mounted) slit into a temperature-controlled chamber, where it is reflected by a small plane mirror into an auto-collimating system comprising a Zeiss-

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1) ten Bruggencate, P. and Jager, F.W.; Veroff.d.Univ.Sternw. Göttingen, No.101, 1951.
2) ten Bruggencate, P. and Voigt, H.H.; ibid., No.122, 1958.

meniscus lens of 16 cm aperture and 8 m. focus, and a Bausch and Lomb (BL) plane grating which is mounted upon a turntable. By varying the angle of rotation of the grating, light from any selected wavelength region and spectral order can be made to return through the lens and be brought to a focus in the plane containing the photographic plate, as illustrated in the diagram (Fig.8.). The grating has a ruled surface of 102 mm. × 127 mm., with 600 lines/mm., and its 2nd order is especially suitable for observations in the near infrared since with the existing auto-collimation arrangement the blaze-angle corresponds in this case to a wavelength of 8300A, the half-width of the blaze region (inside which about 60% of the incident light is reflected) being 2400A. The optical properties of the BL grating have been described in a paper by von Alvensleben[1]. The grating angle (in degrees) and focus of the spectrograph (in relative units with 1 mm. intervals) can be read through an eyepiece and varied by means of electronic controls: the values of both these quantities for a given wavelength and spectral order are easily found from a chart.

2. The Guiding.

Much of the guiding of the solar image on the slit was

1) von Alvensleben, A.: Zeits.f.Ap., 43, 63, 1957.

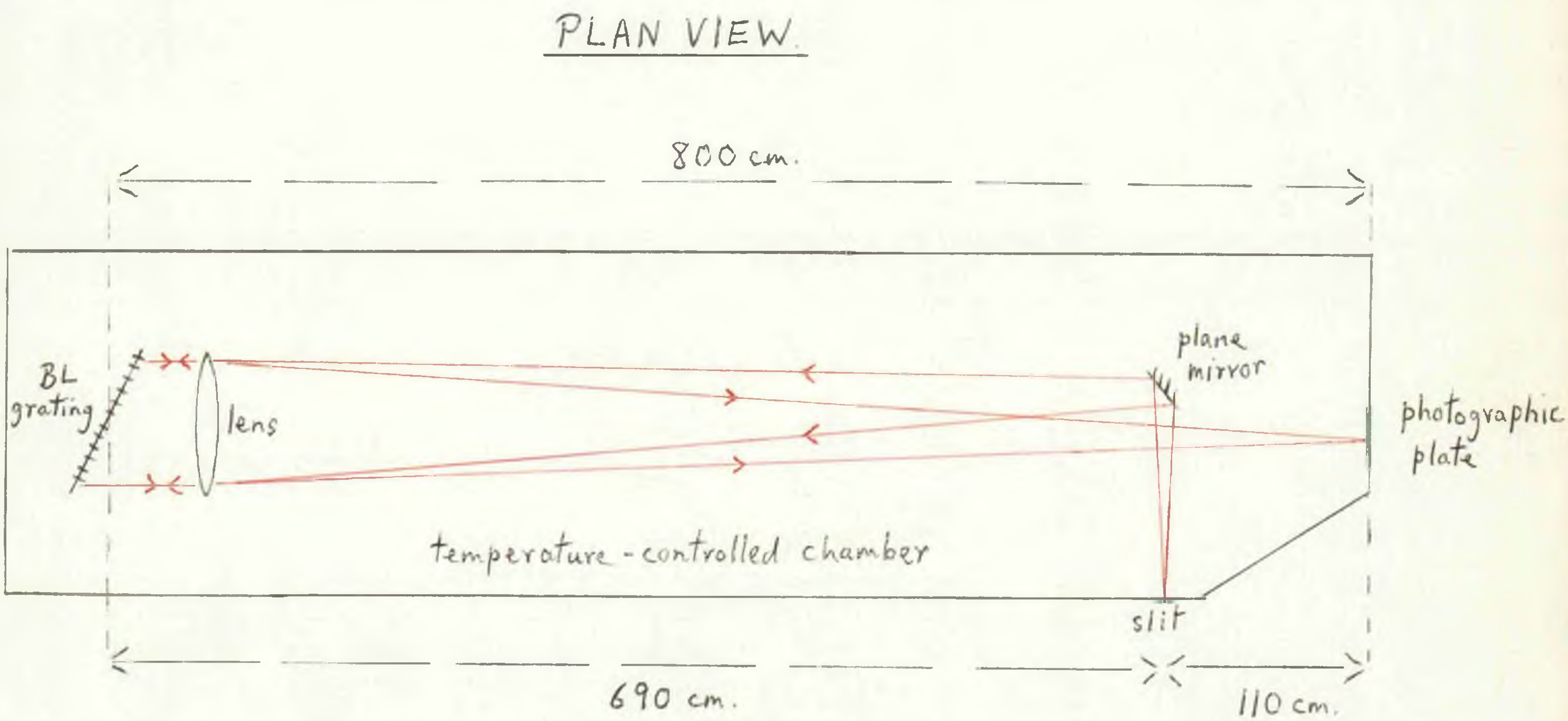


FIG.8: Schematic Representation of Spectrographic Apparatus employed for Infrared Observations (not to scale).

done indirectly, by setting the limb of an auxiliary image of 190 mm. diameter with reference to a Cartesian co-ordinate system (origin at disk centre) on a sheet of mm-ruled graph paper pinned to a board that can be adjusted by two screws so that the auxiliary image is centred on a circle (representing the solar image) drawn on the graph paper, when the primary image is centred exactly upon a point on the slit. For this purpose, a well defined feature on the Sun's disk - generally a small Sun-spot - served as a suitable reference point. The orientation of the solar image was ascertained by observing the direction of drift of the Sun-spots across the paper. Normally, the axes of the coelostat mirrors lie in the plane of the meridian, and the direction of the slit corresponds to the East-West diameter of the solar image; in this case, the angle between the tangent to the Sun's limb at the ends of its polar diameter and the slit is equal to the position angle P, whose mean value was $26^{\circ}.2$ for the days on which the plates were taken. Only on one occasion, for the plate IR2 (cf. Table 11), the azimuth of the coelostat mirrors was changed by 8° in order to prevent the shadow of the secondary mirror from falling upon the primary mirror, but the system was moved back to its usual position after the first four comparisons (i.e. those for the four selected limb positions nearest to the centre of the disk) had been made; the distances along the polar radius being carefully chosen so as to conform with the disk positions used in the Arcetri observations (Table 5).

The accuracy in setting the limb of the auxiliary image with respect to each of the two orthogonal co-ordinates is about $\pm$ 0.5 mm. under normal seeing conditions: this corresponds to a possible error in positioning the larger image on the slit of about

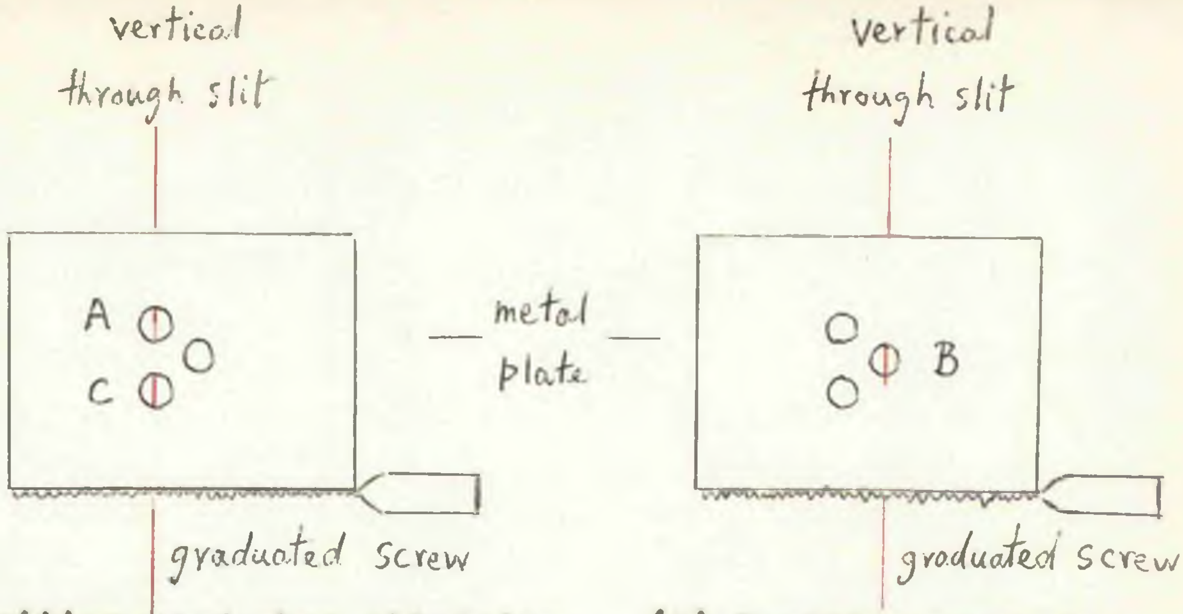
$$\pm\left(\sqrt{2} \times 0.5 \times \frac{220}{190}\right) = \pm 0.8 \text{ mm.}$$

Thus for the three limb positions nearest to the edge of the disk, where we are aware from our study of previous limb effect investigations by other observers that an uncertainty of this order may be critical to the results obtained, the image was guided directly on the slit with reference to scratches made on the metal plate; the estimated accuracy in this case is of the same order as for the Arcetri observations: viz. to within $\pm$ 0.5 mm. Due care was always taken to check from time to time that the image was focussed as exactly as possible in the plane of the slit, since this is another source of uncertainty in defining the limb positions which might become significant at the extreme limb; even so, it is thought that the overall accuracy attainable is limited as much by the heating effect of the Sun's rays on the optical system as by the atmospheric scintillation, so any improvement resulting from better seeing would be slight.

3. Description of the Infrared Limb Effect Observations.

In the 2nd order of the BL grating, spectral comparisons were made on the Ilford Long Range Spectrum plates*) between the central portion of the Sun's image and the 9 disk positions given in Table 5. This was done in a manner analagous to that used at Arcetri, except that the function of the "V" was fulfilled by a metal plate in which three holes (each approximately 1 mm. wide) had been drilled (Fig.9). With the plate in the position illustrated in Fig.9a, and the Sun's image positioned so that its centre lay between the two outer holes, light entered the spectrograph simultaneously from the portions of the slit A and C; the spectra so formed were taken to be representative of the disk centre. By turning a graduated screw to move the metal plate into the position shown in Fig.9b, and centering the Sun's image so that light from a specified limb position was entering the slit at B, a spectrum was photographed between the two centre spectra on the photographic plate, as indicated by the letters A, B and C in Fig.10. Since the magnification of the spectrograph is unity, all three spectra thus obtained

*) It was necessary to cut the 12cm×9cm platés ordered for the Arcetri programme in halves i.e.to 6cm×9cm, so that they fitted into the plate holders at Göttingen.



(a) Position used when obtaining centre-of-disk spectra.

(b) Position used when obtaining limb spectra.

FIG.9: Diagrams illustrating Method of Spectral Comparison.

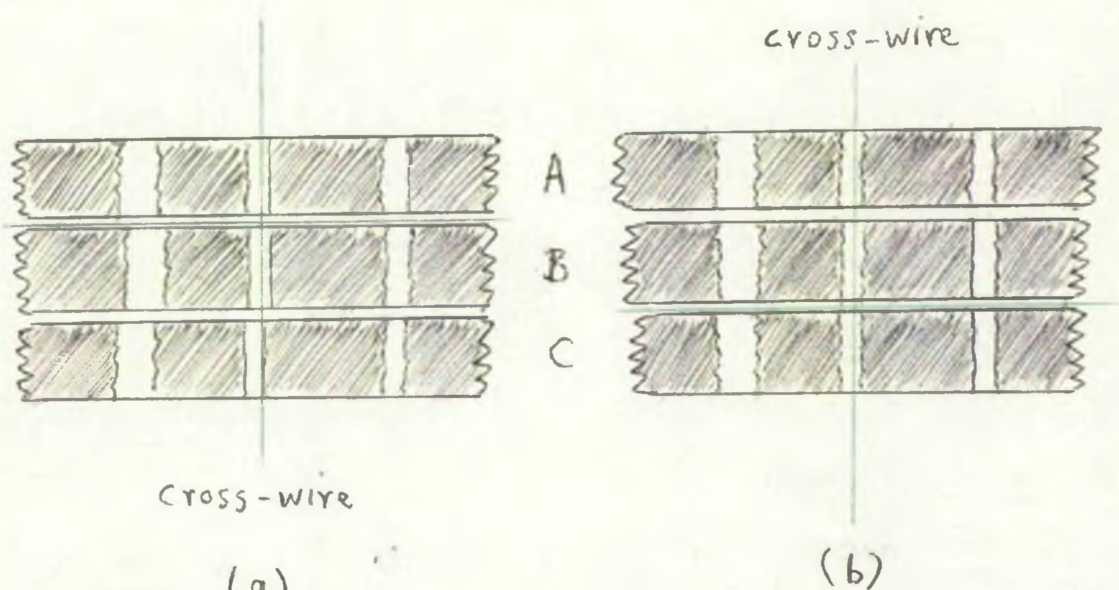


FIG.10: Diagram illustrating Method of Measurement.

were 1 mm. wide (the same as the diameter of the holes in the metal plate through which the light had entered the spectrograph). Between the two exposures a shutter immediately behind the slit was closed by moving a small lever, in order to block light from the spectrograph while the metal plate was being moved. The photographic plate was not disturbed in any way until both exposures were made and the shutter again closed, after which the plate carrier was lowered by 5 mm. and the procedure repeated for another disk position.

The precise wavelength region selected for this investigation is that centred near 7770A, which is of special interest since it contains the OI triplet and the two NiI lines on which Voigt has based his Three-Stream Model of the Sun's photosphere[1]. The linear dispersion at this wavelength in the 2nd order was found to be about 0.92 A/mm., which is appreciably higher than that obtainable with the Arcetri equipment; on the other hand, the theoretical resolving power is only 600 × 2 × 127 = 152,400. An RG8 Shott filter was always used to eliminate interference from the higher spectral orders: the exposure times required with a slit width of 0.043 mm.[*] varied between 30 secs. for the

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1) Voigt,H.H.; Zeits.f.Ap.,<u>40</u>,157,1956; ibid.<u>47</u>,144,1959.
*) This slit-width was chosen since it gives the same instrumental profile at 7770A as a slit 0.030 mm. wide in the green near 5300A.

disk centre up to 120 secs. for the extreme limb exposures, less than one-tenth of that which would have been required with the Arcetri spectrograph.

From a total of 14 plates collected during a favourable spell of weather in October 1959, 10 - five containing North limb comparisons and the other five South limb comparisons along the polar diameter - were chosen for reduction; particulars relating to these are listed in Table 11. A description of the infrared Fraunhofer lines selected for measurement is given in Table 12, the sources of information being the same as before (cf. p.101). In addition to the OI and NiI lines for which the intensity profiles have been measured by Voigt[1], the table includes 3 FeI lines to supplement the list in Table 10.

1) Voigt,H.H.; Zeits.f.Ap.,40,157,1956; ibid.47,144,1959.

TABLE 11.

Particulars of Plates centred near 7770A.

Plate Desig.	Date	Continental Time (CT)	P°	Limb Observed	Atmospheric Conditions
IR 1	2.10.59	8.50 - 9.10	26.06	North	Clear
IR 2	4.10.59	14.00 - 14.35	26.18	"	Hazy
IR 3	4.10.59	14.40 - 15.05	26.18	South	"
IR 4	5.10.59	10.25 - 11.00	26.23	"	"
IR 5	5.10.59	14.40 - 15.10	26.23	North	"
IR 6	5.10.59	15.15 - 15.50	26.23	South	Clear
IR 7	5.10.59	15.55 - 16.30	26.23	North	"
IR 8	6.10.59	8.15 - 8.45	26.27	South	"
IR 9	10.10.59	9.35 - 10.10	26.35	"	Hazy
IR 10	12.10.59	10.25 - 11.00	26.35	North	"

TABLE 12.

Description of Lines Measured on the Göttingen Plates.

λ(A)	$\frac{W}{\lambda}\times10^6$	El.	Mult. No.	Ex. Pots. Low	Ex. Pots. High	J	Mult. Desig.
7748.281	12.9	FeI	402	2.94	4.53	5-4	$b^3G-y^3F^o$
7751.18	5.7	FeI	1304	4.97	6.56	5-4	$x^5F^o-h^5G$
7771.96	8.0	O I	1	9.11	10.69	2-3	$3^5s^o-3^5P$
7774.18	7.2	O I	1	9.11	10.69	2-2	
7775.40	6.2	O I	1	9.11	10.69	2-1	
7780.586	14.4	FeI	1154	4.45	6.04	3-2	$z^3G^o-e^3F$
7788.95	10.8	NiI	62	1.94	3.53	1-2	$a^3P-z^3P^o$
7797.62	9.2	NiI	201	3.88	5.46	2-2	$z^1D^o-e^1D$

CHAPTER IV.

THE MEASUREMENT AND REDUCTION OF THE OBSERVATIONAL DATA COLLECTED AT THE ARCETRI AND GÖTTINGEN OBSERVATORIES.

1. The Method of Measurement.

It was originally intended that the reduction of the Arcetri plates should be made with the "Mioni" spectral-comparator described by Righini[1]; however, a re-examination by the writer of its periodic and progressive screw errors[2] led to the realisation that these were so large as to render this instrument unsuitable for precise wavelength determinations. In view of this negative result, a similar investigation was carried out on the Abbé-Zeiss Comparator (No.658) at the Universitäts-Sternwarte, Göttingen.

For the purpose of testing the periodic screw error of the latter instrument, two fine scratches made on the emulsion side of a developed photographic plate, separated by approximately one-half pitch of the screw, served as

1) Righini, G.; Publ.Arcetri, No.47, 1930.
2) Forbes, E.G.; Publ.Arcetri, No.33, 1959; Mem.della Soc. Astr.Ital., 30, 333, 1960.

suitable reference marks. Four independent determinations of the difference between these marks were made at equal intervals of 0.1 rev. (= 35μ) over one complete revolution; the mean of these 11 positions was found, and the residuals obtained by subtracting the mean from each of the observed values. This procedure was repeated at equal intervals along the 14 mm. long screw: viz. at the beginning (0) and after 9, 18, 27, 36 revs. respectively, the total length corresponding to 40 revs. Since the residuals showed no systematic variation with the part of the screw being tested, mean values were found by averagining over the five different settings. The final results obtained in this way are shown in Fig.11 along with their probably errors: they exhibit no tendency to vary periodically throughout the revolution, and in every case are less than ±1μ. The average error for all 11 positions is exactly ±0.4μ - less than the standard deviation of each - hence the periodic error, if it exists, is so small that it lies within the error of setting. From the above figures we conclude that no systematic correction for periodic error need be applied to the measurements made with the Abbé-Zeiss comparator; furthermore, since we are interested in obtaining values for the relative wavelength differences only, without reference to standard laboratory or atmospheric lines, there was no

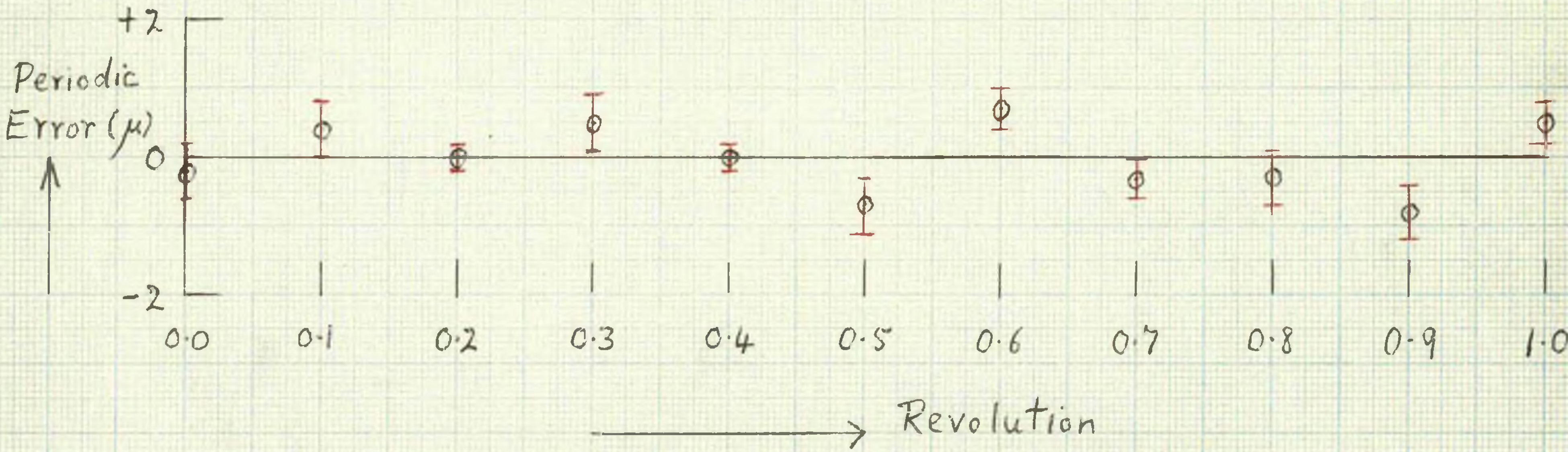

FIG.11: The Periodic Error of the Abbé-Zeiss Comparator at Göttingen.

need to study the form of the progressive error, nor to specify the part of the screw at which the measurements have been made. This instrument was consequently used to reduce the entire observational data; with its help, the measurements of the relative displacements of the Fraunhofer lines listed in Tables 10 and 12 were made simply as follows:-

Each photographic plate was placed in the rotatable carrier attached to the comparator and held in position by metal clips, then rotated into such a position that, when the carfier was moved in the direction of the screw, the centre of the cross-wire traced out the line of dispersion. The cross itself was properly orientated by rotating the eyepiece of the viewing microscope until one of its arms appeared to be parallel to the spectral line. In the case of the Arcetri spectrograms, the other arm co-incided with the line of dispersion; in the case of the Göttingen spectrograms it did not, since the slit had been originally adjusted so as to be parallel to the rulings on a concave grating also installed in the temperature-controlled chamber, and it so happens that these are not quite in line with the rulings on the BL grating. With the cross set as in Fig. 10a, five settings on the estimated line centre were made first in spectrum A, then in B; and the mean difference *)

*) This should be regarded as a visual average taken over a length of roughly 0.5 mm. of an unresolved zig-zag line structure.

found. The plate was then raised until the cross appeared in the position shown in Fig.10b, and the same procedure was repeated for the spectra B and C. By averaging the two means, any error due to incorrect orientation of the cross-wire should be eliminated. This was done for all 9 comparisons on the same plate, by moving the carrier up or down as the case may be; after which the plate was reversed, the measurements repeated, and another set of mean values obtained, in order to compensate any physiological tendency of the eye to over- or under-estimate the "true" position of the line centre. All the measurements were made by approaching the spectral lines from the left side (as viewed through the microscope), which corresponds to the forward direction of the screw; by so doing, the effect of "back-lash" was avoided. Despite this precaution, it was characteristic of most of the measurements on the infrared lines that the values of the relative differences obtained when the plate was viewed in the direction of increasing wavelength tended to be systematically higher than those found with the plate in the reverse direction, the differences often amounting to several microns. In extreme cases the measurements were repeated, but similar results were nearly always obtained, proving that the discrepancy was systematic (and not wholly accidental) in character; since the measurements made on the Arcetri spectrograms exhibited no such effect, it is thought to arise as a result of the lack of alignment between the BL grating and the slit, mentioned above. This should have no appreciable influence on the final figures, however, as these were obtained by averaging both sets of mean values.

2. The Consistency and General Reliability of the Measured Line Shifts.

The results obtained from these micrometer measurements were converted into wavelength units by multiplying by the reciprocal dispersion - this being taken as 1.47 A/mm. and 0.92 A/mm. for the Arcetri and Göttingen plates respectively. A repetition of the measures was generally found to yield values differing by ±2mA or less from those initially obtained, so this figure may be regarded as a direct indication of the consistency of the method; a higher degree of internal accuracy is scarcely to be expected in view of the general difficulty associated with visually estimating the centre of an intrinsically broad Fraunhofer line. Two sets of independent determinations of the shifts of the FeI line λ5324.2 are quoted in Table 13: an inspection of the 9 × 4 = 36 plate values shows that 31 were reproduced to within ±2mA, while of the nine pairs of mean values only one

TABLE 13.

Results of Measurements on the FeI line λ5324.2

Relative Wavelength Shifts (mA) measured in February 1960

Disk Pos. / Plate	1	2	3	4	5	6	7	8	9
G1	+2	+1	+3	-2	+4	+5	+7	+5	+10
G2	+2	-2	0	+3	-1	-3	+9	+7	+ 5
G3	0	0	-1	-2	-1	+1	+3	+4	+ 9
G4	+1	+2	0	+9	+10	+2	+5	+7	+13
Mean	+1.3	+0.3	+0.5	+2.0	+3.0	+1.3	+6.6	+5.8	+9.3

Relative Wavelength Shifts (mA) measured in July 1960

Disk Pos. / Plate	1	2	3	4	5	6	7	8	9
G1	0	0	+3	-1	+1	+4	+8	+5	+8
G2	+2	0	+2	+3	0	-1	+11	+6	+6
G3	-1	-2	-1	-3	-1	+1	+4	+5	+13
G4	+5	+5	-1	+7	+7	+4	+6	+6	+12
Mean	+1.5	+0.8	+0.8	+1.5	+1.8	+2.0	+7.3	+5.5	+9.8

(at cos θ = .54) differed by more than $\pm$1mA. The scatter among the individual plate values, however, is seen to be considerably in excess of the accidental errors involved, which implies the existence of appreciable systematic errors: these could partly be introduced through comparing spectra of differing photographic densities, or may be due to other causes such as small displacements of the apparatus between consecutive exposures (belonging to the same comparison) and the "bunching" of the photographic grains. Moreover, the possible effect of granulation in producing a small differential shift between spectra of neighbouring disk positions cannot be disregarded, for this might cause the mean wavelength of one of the centre spectra to differ slightly from that of the other, thereby giving rise to small spurious shifts in the results obtained from different plates. Perhaps it is worth mentioning that the systematic displacements associated with the solar rotation were deliberately avoided by confining all observations to the polar diameter.

Another means of checking the reliability of the measures is provided by the existence on the set of spectrograms centred near 6300A of two measurable lines (λ6298.5 and λ6302.0) known to originate in the Earth's atmosphere; for in contradistinction to the Fraunhofer lines, telluric lines should exhibit no systematic centre-limb wavelength variation if this phenomenon is truly of solar origin.

Some insight into the relative importance of the above sources of error was gained from measurements made on control plates containing 9 centre-centre comparisons covering a wide range of photographic density, which indicated that as far as the observations of moderately intense lines are concerned, this latter factor was less liable to produce significant discrepancies than the other causes mentioned. Such a view is supported by the fact that the values found for the relative shifts of the 8 selected lines at the extreme limb position on IR1 were considerably in excess of the corresponding figures obtained from the other nine infrared plates measured, and have consequently been discarded. It is thought that this anomaly can be attributed entirely to a small mechanical displacement of the plate holder after its cover had been pulled aside rather violently at the beginning of the first comparison (which referred to disk position 9). Recent observations made with the very high resolution equipment at Potsdam[1] and at Oxford[2,3] yield further evidence of a considerable spread among the individual plate values, confirming that this feature is inherent in all solar wavelength measurements and must be

1) Schröter, E.H.; Monatsber.d.Dt.Akad.d.Wiss.,Berlin,1,738,1959
2) Adam, M.G.; M.N., 119, 460, 1959.
3) Higgs, L.A.; M.N., 121, 421, 1960.

related principally to the turbulent conditions prevailing in the solar atmosphere, and not so much to the presence of systematic errors in the observational data. The same fundamental difficulty confronting those seeking a satisfactory explanation of the solar red shifts was fully appreciated a quarter of a century ago by Evershed, who believed that owing to the combined effect of localised currents parallel and radial to the Sun's surface, "only mean values from at least 20 spectra can give reliable results" (M.N., 96, 158, 1936).

In the present investigation, evidence that appreciable errors are nevertheless involved in the observational procedure which have nothing to do with the effects of gaseous motions in the Sun's amosphere, is provided by the results of measures made on two telluric lines (λ6298.5 and λ6302.0) contained on the set of spectrograms centred near 6300A; yet although the individual plate values show a considerable scatter about the theoretically expected zero value (Table 14), it is encouraging to note that only one of the nine mean shifts obtained for the line λ6298.5 from the reduction of plates R5-8 is greater than ±2mA. A comparison between the corresponding mean values found for λ6302.0 (which possess a similar degree of consistency) and those derived

TABLE 14.

Results of Measurements on Atmospheric Lines
(4 plates only)

λ6298.5 Limb-Centre Shifts (mA)

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	+5	+1	-1	0	+1	+3	+2	-1	+1
R6	+3	-1	+2	-1	+2	+4	+2	-4	+1
R7	+2	+3	-1	+1	+6	-5	-2	+2	+2
R8	-1	+1	-1	0	-2	0	+1	-2	-2
Mean	+2.3	+1.0	-0.3	0.0	+1.8	+0.5	+0.8	-1.3	+0.5

λ6302.0 Limb-Centre Shifts (mA)

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	+7	+6	-10	-4	0	+7	-1	+5	-1
R6	+4	-6	- 1	-1	0	-3	+5	-3	-2
R7	-3	-4	+ 2	-1	-1	+1	+4	-5	-5
R8	+3	-1	- 4	0	-1	-3	-3	-4	-1
Mean	+2.8	-1.3	-3.3	-1.5	-0.5	+0.5	+1.3	-1.8	-2.3
Mean from R1-10	-1.6	+1.3	-1.5	-0.4	-0.8	+0.5	+3.0	+0.9	-1.1

from all ten plates R1-10 suggests that although the scatter tends to be reduced by basing the results on a greater number of plates, the improvement is too slight to merit the time and labour that would be required if this were done for every line included in the programme; consequently, the majority of these lines were measured on four plates only. In view of the lower degree of consistency which characterised the plate values obtained from the Göttingen observations, the infrared lines were all chosen for additional measurement[*)] three rather strong lines on the Arcetri plates: viz. λ4442.3, λ4447.7, and λ5324.2, whose mean centre-arc shifts have been obtained from recent interferometric observations made at Oxford[1,2)], were similarly treated. Full details of the limb-centre shifts deduced from our own investigation are contained in the following section: the probable errors of the mean values have little significance since the scatter cannot be regarded as being principally due to errors of accidental character, and have consequently not been given.

*) A small error in setting the angle of rotation of the BL grating caused the NiI line λ7797.62 not to appear on the plate IR9; hence its mean values are based on nine plates, while those of the other seven infrared lines refer to all ten plates.

1)Adam, M.G.: M.N., **118**, 106, 1958.

2) Nichols, S. and Clube, S.V.M.: M.N., **118**, 496, 1958.

3. Results of Measurements of the Limb-Centre Shifts of Selected Solar Spectral Lines contained on the Arcetri and Göttingen Spectrograms.

Limb-Centre Shifts (mA) of λ4484.2

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
B1	-1	-3	-2	+4	+5	+1	0	+8	+8
B2	+2	+1	-4	+3	+3	+3	-2	+4	+10
B6	-2	0	+2	+3	+1	+2	+2	+1	+4
B7	-3	-2	+2	0	-2	-1	+4	-4	-2
Mean	-1.0	-1.0	-0.5	+2.5	+1.8	+1.3	+1.0	+2.3	+5.0

Limb-Centre Shifts (mA) of λ4389.2

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
B1	0	-4	-1	-2	+3	-1	0	0	+3
B2	+1	-2	-3	-3	+1	+2	-5	0	0
B6	-1	-2	+4	-1	+2	-2	-3	-2	+6
B7	0	-2	0	-7	-3	-8	-1	-4	-6
Mean	0.0	-2.5	0.0	-3.3	+0.8	-2.3	-2.3	-1.5	+0.8

Limb-Centre Shifts (mA) of λ4442.3

Plate \ Disk Pos	1	2	3	4	5	6	7	8	9
B1	0	-1	+2	+2	+2	+6	+3	+5	+11
B2	-1	-2	-1	+2	-1	+7	-3	+3	+5
B3	-5	-4	-1	+6	-4	+5	+3	+5	+2
B4	-4	+2	+7	-1	-2	+1	+1	+7	+7
B5	0	-2	+3	+5	-2	+5	+3	+6	+7
B6	-1	-3	+3	+1	+2	+2	+3	+3	+6
B7	-1	-1	+1	-3	-2	-3	-1	-1	-2
B8	+1	-3	-4	-1	-1	-3	+3	+5	+3
B9	+3	0	-1	-1	-3	-6	+3	0	+6
B10	-1	-3	-2	-2	0	-1	+1	0	+6
Mean	-0.9	-1.7	+0.7	+0.8	-1.1	+1.3	+1.6	+3.3	+5.1
Mean of B1,2,8,7	-0.8	-1.8	+1.3	+0.5	+0.3	+3.0	+0.5	+2.5	+5.0

Limb-Centre Shifts (mA) of λ4447.7

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
B1	-2	-2	-1	+2	+2	+4	0	+5	+5
B2	+1	0	-3	+2	+1	+4	+3	+2	0
B3	-2	-5	+1	+4	0	+5	+3	+1	+5
B4	-4	+1	+5	-1	-3	+3	+3	+7	+2
B5	+1	-1	+4	+5	-3	+4	0	+5	+1
B6	0	-1	+2	+6	+3	-1	+3	+2	+3
B7	0	0	-1	+1	+1	-3	+1	-4	-4
B8	+1	-2	-5	-1	0	-5	+3	-1	+3
B9	+1	+1	-3	0	-4	-5	+4	+1	+3
B10	+1	-2	-1	-2	-1	0	+1	-1	-1
Mean	-0.3	-1.1	-0.2	+1.6	-0.4	+0.6	+2.1	+1.7	+1.7
Mean of B1,2,6,7	-0.3	-0.8	-0.8	+2.8	+1.8	+1.0	+1.8	+1.3	+1.0

Limb-Centre-Shifts (mA) of λ4485.7

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
B1	-1	+3	0	0	+1	+5	+4	+8	+4
B2	-2	+3	0	-3	0	+7	+1	+5	+2
B6	+2	-5	+1	+4	0	0	+3	+1	+5
B7	+1	+2	-3	-3	-3	-4	+1	-3	-3
Mean	0.0	+0.8	-0.5	-0.5	-0.5	+2.0	+2.3	+2.8	+2.0

Limb-Centre Shifts (mA) of λ4489.7

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
B1	0	+1	-1	+2	+3	+5	0	+6	+6
B2	-1	0	-3	+3	+3	+3	-5	+5	-2
B6	0	-3	-1	+4	0	-1	+2	0	+3
B7	-1	+2	+1	-5	-2	-6	0	-5	-7
Mean	-0.5	0.0	-1.0	+1.0	+1.0	+0.3	-0.8	+1.5	0.0

Limb-Centre Shifts (mA) of λ5247.1

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	0	+3	+5	-8	+1	+1	+3	+2	+3
G2	-1	-3	-2	0	-2	-9	-4	0	0
G3	0	-5	-3	-4	-2	-3	+1	0	+1
G4	+7	0	-2	+7	0	-1	0	-2	+8
Mean	+1.5	-1.3	-0.5	-1.3	-0.8	-3.0	0.0	0.0	+3.0

Limb-Centre Shifts (mA) of λ5250.2

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+1	-2	+4	-5	0	-1	+2	+1	+6
G2	-4	+1	+1	-2	-4	-7	-2	+4	+2
G3	0	-6	-2	-1	-5	-4	-2	-2	0
G4	+4	+2	-4	+4	0	0	-1	-2	+7
Mean	+0.3	-1.3	-0.3	-1.0	-2.3	-3.0	-0.8	+0.3	+3.8

Limb-Centre Shifts (mA) of λ5322.0

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+2	-2	+2	-10	-2	0	+3	-1	+6
G2	+3	+1	-2	+4	0	-3	-1	+3	0
G3	-2	-3	0	-4	-3	-1	-1	+5	+3
G4	+4	0	-2	+6	-1	+3	+4	0	+7
Mean	+1.8	-1.0	-0.5	-1.0	-1.5	-0.3	-1.3	+1.8	+4.0

Limb-Centre Shifts (mA) of λ5364.9

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+5	-1	+2	-4	+1	+3	+5	+3	+7
G2	0	-1	0	+1	+3	-6	+6	+6	+4
G3	-2	-8	+2	-1	-2	-1	+3	+8	+3
G4	+5	+1	-2	+7	+7	+1	+7	+4	+12
Mean	+2.0	-2.3	+0.5	+0.8	+2.3	-0.8	+5.3	+5.3	+6.5

Limb-Centre Shifts (mA) of λ5324.2

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	0	0	+3	-1	+1	+4	+8	+5	+8
G2	+2	0	+2	+3	0	-1	+11	+6	+6
G3	-1	-2	-1	-3	-1	+1	+4	+5	+13
G4	+5	+5	-1	+7	+7	+4	+6	+6	+12
G5	-2	-1	+2	-1	+2	+2	+13	+9	+14
G6	+2	+2	+3	-3	+1	+4	+9	+5	+11
G7	+4	+1	+2	-3	+3	+4	+9	+11	+5
G8	-5	-6	-5	-6	-3	-1	+8	+5	+2
G9	-1	-2	+7	-5	-1	+9	+10	+8	+7
G10	+4	-1	+1	+4	+2	+4	+4	+5	+5
Mean	+0.8	-0.4	+1.3	-0.8	+1.1	+3.0	+8.2	+6.5	+8.3
Mean of G1,2,3,4	+1.5	+0.8	+0.8	+1.5	+1.8	+2.0	+7.3	+5.5	+9.8

Limb-Centre Shifts (mA) of λ5365.4

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+3	-1	0	0	+2	+4	+2	+1	+5
G2	0	-3	-3	+4	-1	-1	+3	+3	+2
G3	-3	-5	-3	-3	-5	0	0	+1	+5
G4	+5	+1	-3	+8	+1	+1	+6	+5	+9
Mean	+1.3	-2.0	-2.3	+2.3	-0.8	+1.0	+2.8	+2.5	+5.3

Limb-Centre Shifts (mA) of λ5367.5

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+4	+1	+5	0	+2	+4	+7	+1	+6
G2	-1	-4	-1	+2	0	-2	+3	+4	+5
G3	0	-4	0	-3	-1	-3	0	+5	+4
G4	+3	+4	+1	+4	+1	+2	+3	+3	+10
Mean	+1.5	-0.8	+1.3	+0.8	+0.5	+0.3	+3.3	+3.3	+6.3

Limb-Centre Shifts (mA) of λ5379.6

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+6	0	+4	-8	0	0	+1	+1	-2
G2	-1	0	-1	+6	-4	-6	+8	+2	0
G3	+1	-7	+4	-4	-5	-2	-2	+9	+2
G4	+2	+3	-4	+5	+1	+4	+5	+1	+10
Mean	+2.0	-1.0	+0.8	-0.3	-2.0	-1.0	+3.0	+3.3	+2.5

Limb-Centre Shifts (mA) of λ5383.4

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
G1	+2	-1	+2	-3	+5	+6	+5	+3	+6
G2	0	+2	-1	0	+1	-4	+5	+3	+1
G3	+2	-4	+3	+1	-2	+2	-1	+3	+5
G4	+3	+3	-3	+7	+7	+1	+9	+5	+8
Mean	+1.8	0.0	+0.3	+1.3	+2.8	+1.3	+4.5	+3.5	+5.0

Limb-Centre Shifts (mA) of λ6056.0

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
01	+2	+3	+3	-1	-2	-3	-4	+1	+3
02	-4	0	-2	-1	-1	+3	+3	+6	+3
03	+3	-2	+7	+3	+2	-2	+5	+2	-3
04	0	+1	0	-3	-5	+2	+4	+2	0
Mean	+0.3	+0.5	+2.0	-0.5	-1.5	0.0	+2.0	+2.8	+0.8

Limb-Centre Shifts (mA) of λ6065.5

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
01	+1	-1	+2	-1	+5	-3	+1	0	+4
02	-3	+2	+1	+1	-2	+6	+2	+2	+2
03	+1	+1	+1	-4	0	0	+5	+4	+2
04	-3	+1	+2	-2	-1	-6	+1	-1	+3
Mean	-1.0	+0.8	+1.5	-1.5	+0.5	-0.8	+2.3	+1.3	+2.8

Limb-Centre Shifts (mA) of λ6137.0

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
01	-1	+2	+1	-2	-1	-3	-3	0	+10
02	-2	+5	+1	+5	0	+6	+6	+3	+4
03	+1	+1	+1	0	-2	+6	+6	+1	+10
04	-6	+4	+4	-2	0	-8	+1	+4	+4
Mean	-2.0	+3.0	+1.8	+0.3	-0.8	+0.3	+2.5	+2.0	+7.0

Limb-Centre Shifts (mA) of λ6151.6

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
01	-3	-1	-2	0	0	-5	0	-5	-1
02	+6	+1	-8	+8	-2	+2	-2	+11	0
03	+3	+5	+6	-5	+4	0	+3	0	+2
04	-5	-3	+8	-7	+4	-4	-6	+2	-1
Mean	+0.3	+0.5	+1.0	-1.0	+1.5	-1.8	-1.3	+2.0	0.0

Limb-Centre Shifts (mA) of λ6157.7

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
01	+4	-2	+3	-3	-1	+3	-1	+1	+10
02	+1	0	-1	+2	+5	+2	+4	+1	+5
03	+1	-1	+8	+1	-2	-1	+6	+1	+2
04	-4	-6	+7	+3	-2	-4	+3	+5	+1
Mean	+0.5	-2.3	+4.3	+0.8	0.0	+0.0	+3.0	+2.0	+4.5

Limb-Centre Shifts of λ6173.3

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
01	0	-2	-3	-2	+5	-5	-5	-2	-2
02	-9	+3	-3	+1	-1	+4	+1	+3	0
03	0	-2	+4	-2	-2	0	-1	+2	+1
04	-6	+5	0	-5	-3	0	+1	+3	-1
Mean	-3.8	+1.0	-0.5	-2.0	-0.3	-0.3	-1.0	+1.5	-0.5

Limb-Centre Shifts (mA) of λ6246.3

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	+1	+5	-1	+2	0	0	+2	+7	+9
R6	+1	-2	-7	-3	+1	0	+4	+2	+7
R7	+1	+3	-5	+2	+11	+3	+4	+7	+9
R8	+3	0	0	+1	-1	+3	+2	+5	+10
Mean	+1.5	+1.5	-3.3	+0.5	+2.8	+1.5	+3.0	+5.3	+8.8

Limb-Centre Shifts (mA) of λ6265.1

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	-1	+1	0	-2	+1	-2	-3	+2	+8
R6	+3	-2	-2	-3	-1	0	+4	-1	+9
R7	+5	+2	-2	+1	+9	0	-1	+6	+6
R8	-1	-2	-3	0	-6	0	-3	+4	+7
Mean	+1.5	-0.3	-1.5	-1.0	+0.8	-0.5	-0.8	+2.8	+7.5

Limb-Centre Shifts (mA) of λ6297.8

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	0	+3	-1	+2	+3	-3	-3	+4	+4
R6	-2	-2	-3	-1	-2	-5	+4	+4	+2
R7	0	+3	+4	-2	+9	+3	+2	0	+8
R8	+1	+4	+2	+1	+5	+2	-5	+2	+7
Mean	-0.3	+2.0	+0.5	0.0	+3.8	-0.8	-0.5	+2.5	+5.3

Limb-Centre Shifts (mA) of λ6301.5

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	+1	+3	-2	0	+3	0	-2	+3	+9
R6	-3	+4	-5	+4	+5	+1	+8	+2	+9
R7	0	-1	+1	-1	+11	+7	+4	+8	+11
R8	+3	+1	0	+2	-3	-1	0	+4	+8
Mean	+0.3	+1.8	-1.5	+1.3	+4.0	+1.8	+2.5	+4.3	+9.3

Limb-Centre Shifts (mA) of λ6335.3

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	-1	-3	-2	+2	-2	-4	+5	+5	+7
R6	-1	-2	-1	+2	+1	-4	+7	+2	+1
R7	+1	+1	-2	0	+9	+2	+5	+1	+9
R8	-1	+1	+2	-1	-1	-1	-3	+1	+13
Mean	-0.5	-0.8	-0.8	+0.8	+1.8	-1.8	+3.5	+2.3	+7.5

Limb-Centre Shifts (mA) of λ6358.7

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
R5	-1	+2	+2	0	-1	0	+2	+3	+9
R6	+3	-3	-3	-2	0	0	+10	+2	+9
R7	+1	+2	-5	0	+6	+2	+2	+7	+11
R8	-1	-3	+2	+2	-1	0	+1	+3	+12
Mean	+0.5	-0.5	-1.0	0.0	+1.0	+0.5	+3.8	+3.8	+10.3

Limb-Centre Shifts (mA) of λ7748.3

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
IR 1	+2	-4	0	-9	+2	+1	+8	+4	/
IR 2	-3	-2	-1	+9	-6	+2	+1	+6	+10
IR 3	+2	+4	+5	+2	+3	+4	0	+9	+5
IR 4	-2	0	-4	+5	+3	+2	+5	+9	+6
IR 5	-1	+1	+6	+7	-1	0	+4	+11	+7
IR 6	-1	+1	+7	+2	+1	+3	+8	+13	+10
IR 7	-4	0	-2	+3	-4	-3	+5	+6	+7
IR 8	-2	+3	0	-3	-1	+2	+5	+3	+9
IR 9	-2	+2	-6	-5	+5	+5	+3	+2	+9
IR10	+1	+3	-7	-1	+10	+3	+8	+3	+4
Mean	-1.0	+0.8	-0.2	+1.0	+1.2	+1.9	+4.7	+6.6	+7.4

Limb-Centre Shifts (mA) of λ7751.2

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
IR 1	+1	-2	-1	-7	-2	-2	+4	+11	/
IR 2	-3	+2	-2	-2	-6	+6	+3	+4	+2
IR 3	+1	+1	+8	+2	-4	+6	+4	+8	+5
IR 4	+3	-2	-4	+2	-2	+2	0	+10	+2
IR 5	+1	-3	+3	+3	+1	-1	-4	+6	+3
IR 6	+2	+4	+2	-8	-11	+5	-3	+10	+7
IR 7	-5	-2	+6	0	0	-1	+4	+3	0
IR 8	-4	-3	+1	-5	-5	+4	-1	+7	+3
IR 9	-5	+3	-2	-5	+4	+1	+6	+1	+11
IR10	-1	+1	-4	-4	+1	+3	+2	+6	0
Mean	-1.0	-0.1	+0.7	-2.4	-2.4	+2.3	+1.5	+6.6	+4.8

Limb-Centre Shifts (mA) of λ7772.0

Plate \ Disk Pos	1	2	3	4	5	6	7	8	9
IR 1	-3	-5	-2	-5	-1	-8	+6	-2	/
IR 2	-2	+3	-2	+8	-1	-1	-2	-4	+3
IR 3	+1	+3	+2	-3	0	+1	+3	+10	+1
IR 4	0	-1	-1	+4	-3	0	+7	-1	-1
IR 5	-1	+1	+4	+6	0	+3	0	+1	+7
IR 6	-3	-5	+2	0	-1	+10	+1	+4	+8
IR 7	-2	-2	+2	+2	-4	+1	+6	-2	+8
IR 8	-1	+7	+3	-1	+3	+3	+4	-1	+5
IR 9	-5	+9	-10	0	+11	+1	+5	+1	-1
IR10	+3	+5	-5	+3	+4	0	+9	-1	+9
Mean	-1.3	+1.5	-0.7	+1.4	+0.8	+1.0	+3.9	+0.5	+4.8

Limb-Centre Shifts (mA) of λ7774.2

Plate \ Disk Pos	1	2	3	4	5	6	7	8	9
IR 1	-1	-5	+2	-9	+2	-2	+4	0	/
IR 2	0	+3	0	0	-2	+6	-6	+3	-1
IR 3	+1	+6	+3	+2	-4	+8	-4	+9	-3
IR 4	+5	+5	+5	+4	0	+7	+3	+8	+1
IR 5	-2	+5	-2	+7	-7	-4	+4	-3	+1
IR 6	+1	-5	+6	-4	+1	+4	+5	+14	+1
IR 7	-10	-2	0	+4	+1	+6	+9	+5	-1
IR 8	+2	+1	-4	-4	-1	-6	+7	+1	+3
IR 9	0	+12	-5	-4	+6	-1	+5	+10	+1
IR10	-4	-1	-5	-1	+3	0	-3	+7	-2
Mean	-0.8	+1.9	0.0	-0.5	-0.1	+1.8	+2.4	+5.4	0.0

Limb-Centre Shifts (mA) of λ7775.4

Plate \ Disk Pos	1	2	3	4	5	6	7	8	9
IR 1	+4	-7	-4	-6	+1	0	+3	+2	/
IR 2	+3	+5	+4	+9	+2	+8	+4	+6	+6
IR 3	+8	+8	+8	+6	-2	+12	0	+13	+7
IR 4	+6	-8	-5	+2	-1	+2	-3	+8	+8
IR 5	+8	-2	+1	+3	+6	+6	+9	+12	+12
IR 6	-11	+2	+4	+7	+4	+3	+14	+15	+8
IR 7	-8	0	+4	+7	-4	+9	+2	+6	-1
IR 8	-4	-6	+5	+2	-7	-2	0	+3	+1
IR 9	+1	+5	-14	+3	+3	+3	+6	-3	+4
IR10	+4	-5	+1	-3	+7	+14	+4	+8	0
Mean	+2.1	-0.8	+0.4	+3.0	+0.9	+5.5	+3.9	+7.0	+5.0

Limb-Centre Shifts (mA) of λ7780.6

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
IR 1	+4	-3	-2	-9	-2	-1	+9	+4	/
IR 2	-1	-1	+1	+7	-2	+1	+4	+8	+5
IR 3	+1	+7	+6	+7	-4	+6	+5	+13	+7
IR 4	+3	+1	-2	+6	+3	+4	+5	+4	+5
IR 5	-1	0	+7	+7	-2	+3	+5	+9	+10
IR 6	-2	-2	+5	-1	0	+5	+7	+9	+2
IR 7	-6	-1	0	+5	-3	-4	+7	+6	+4
IR 8	0	+2	-2	0	+2	+3	+2	+2	+10
IR 9	-2	+4	-3	0	+5	+4	+4	0	+5
IR10	0	+2	-2	-1	+7	+4	+5	+5	+4
Mean	-0.4	-0.9	+0.8	+2.1	+0.4	+2.5	+5.3	+6.0	+5.8

Limb-Centre Shifts (mA) of λ7789.0

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
IR 1	0	-3	-1	-11	0	0	+7	+2	/
IR 2	0	+2	0	+5	-7	+1	+10	+11	+7
IR 3	+4	+1	+3	+2	-3	+3	+2	+11	+6
IR 4	+1	+1	-1	+3	+1	+3	+4	+11	+9
IR 5	0	-1	+2	+4	-4	-2	0	+6	+8
IR 6	-2	0	+4	-2	-5	+4	+6	+5	+5
IR 7	-1	-5	-2	+5	-1	-4	+5	+6	+4
IR 8	-4	+1	-2	-3	-2	+1	+4	0	+7
IR 9	-2	-1	-4	-2	+5	-1	+8	+6	+11
IR10	+3	0	-4	-5	+5	+7	+3	+3	+5
Mean	-0.1	-0.5	-0.5	-0.4	-1.1	+1.2	+4.9	+6.1	+6.9

Limb-Centre Shifts (mA) for λ7797.6

Disk Pos / Plate	1	2	3	4	5	6	7	8	9
IR 1	+1	0	0	-8	0	-1	+11	+8	/
IR 2	0	0	+3	+4	-5	+6	+6	+7	+7
IR 3	+3	+6	+6	+5	0	+4	+2	+13	+4
IR 4	+2	+2	-3	+5	+2	+1	+4	+7	+3
IR 5	-3	-2	+5	+4	+3	0	+1	+8	+5
IR 6	-1	-3	+4	-3	0	+11	+10	+10	+7
IR 7	-5	-5	0	+3	-2	-1	+5	+4	+5
IR 8	-2	+1	+1	-3	-1	+5	+5	+2	+8
IR 9	-	-	-	-	-	-	-	-	-
IR10	+4	+1	-4	0	+6	+5	+5	+7	+5
Mean	-0.1	0.0	+1.3	+0.8	+0.3	+3.3	+5.4	+7.3	+5.5

4. Summary of Results.

For the sake of consistency, the mean values quoted in Table 15 for the 28 FeI lines contained in the Arcetri data are based on 4 plates only. Although the results for the infrared lines in Table 16 are all based on 10 plates, the relative increase in accuracy ($= \frac{\sqrt{10}}{\sqrt{4}}$, if the scatter of the individual plate values is randomly distributed about the arithmetic mean) is exactly*) offset by the relative decrease in linear dispersion. Having established the order of accuracy and the overall reliability of the data summarised in these two tables, we are now in a position to discuss the general features of the solar limb effect emerging from the present investigation in an attempt to gain a little more insight into the origin of this phenomenon.

*) By chance, it happens that $\frac{\sqrt{10}}{\sqrt{4}} = 1.6 = \frac{1.47 \text{ Å/mm}}{0.92 \text{ Å/mm}}$.

TABLE 15.

The Limb Effect of Selected Medium-Intense FeI Lines contained in the Visible Solar Spectrum

$\cos\theta$ / λ(A)	Mean Limb-Centre Shifts (mA) .97	.91	.80	.69	.54	.43	.27	.22	.155
4389.2	0.0	-2.5	0.0	-3.3	+0.8	-2.3	-2.3	-1.5	+0.8
4442.3	-0.8	-1.8	+1.3	+0.5	+0.3	+3.0	+0.5	+2.5	+5.0
4447.7	-0.3	-0.8	-0.8	+2.8	+1.8	+1.0	+1.8	+1.3	+1.0
4484.2	-1.0	-1.0	-0.5	+2.5	+1.8	+1.3	+1.0	+2.3	+5.0
4485.7	0.0	+0.8	-0.5	-0.5	-0.5	+2.0	+2.3	+2.8	+2.0
4489.7	-0.5	0.0	-1.0	+1.0	+1.0	+0.3	-0.8	+1.5	0.0
5247.1	+1.5	-1.3	-0.5	-1.3	-0.8	-3.0	0.0	0.0	+3.0
5250.2	+0.3	-1.3	-0.3	-1.0	-2.3	-3.0	-0.8	+0.3	+3.8
5322.0	+1.8	-1.0	-0.5	-1.0	-1.5	-0.3	-1.3	+1.8	+4.0
5324.2	+1.5	+0.8	+0.8	+1.5	+1.8	+2.0	+7.3	+5.5	+9.8
5364.9	+2.0	-2.3	+0.5	+0.8	+2.3	-0.8	+5.3	+5.3	+6.5
5365.4	+1.3	-2.0	-2.3	+2.3	-0.8	+1.0	+2.8	+2.5	+5.3
5367.5	+1.5	-0.8	+1.3	+0.8	+0.5	+0.3	+3.3	+3.3	+6.3
5379.6	+2.0	-1.0	+0.8	-0.3	-2.0	-1.0	+3.0	+3.3	+2.5
5383.4	+1.8	0.0	+0.3	+1.3	+2.8	+1.3	+4.5	+3.5	+5.0
6056.0	+0.3	+0.5	+2.0	-0.5	-1.5	0.0	+2.0	+2.8	+0.8
6065.5	-1.0	+0.8	+1.5	-1.5	+0.5	-0.8	+2.3	+1.3	+2.8
6137.0	-2.0	+3.0	+1.8	+0.3	-0.8	+0.3	+2.5	+2.0	+7.0
6151.6	+0.3	+0.5	+1.0	-1.0	+1.5	-1.8	-1.3	+2.0	0.0
6157.7	+0.5	-2.3	+4.3	+0.8	0.0	0.0	+3.0	+2.0	+4.5
6173.3	-3.8	+1.0	-0.5	-2.0	-0.3	-0.3	-1.0	+1.5	-0.5
6246.3	+1.5	+1.5	-3.3	+0.5	+2.8	+1.5	+3.0	+5.3	+8.8
6265.1	+1.5	-0.3	-1.5	-1.0	+0.8	-0.5	-0.8	+2.8	+7.5
6297.8	-0.3	+2.0	+0.5	0.0	+3.8	-0.8	-0.5	+2.5	+5.3
6301.5	+0.3	+1.8	-1.5	+1.3	+4.0	+1.8	+2.5	+4.3	+9.3
6302.5	+1.3	+2.5	-0.3	+1.8	+2.5	-2.0	+3.0	+5.5	+8.0
6335.3	-0.5	-0.8	-0.8	+0.8	+1.8	-1.8	+3.5	+2.3	+7.5
6358.7	+0.5	-0.5	-1.0	0.0	+1.0	+0.5	+3.8	+3.8	+10.3

TABLE 16.

The Limb Effect of Selected Medium-Intense Fraunhofer Lines contained in the Near-Infrared Solar Spectrum.

Mean Limb-Centre Shifts.

λ(A)	El \ $\cos\theta$	.97	.91	.80	.69	.54	.43	.27	.22	.155
7748.3	FeI	-1.0	+0.8	-0.2	+1.0	+1.2	+1.9	+4.7	+6.6	+7.4
7751.2	FeI	-1.0	-0.1	+0.7	-2.4	-2.4	+2.3	+1.5	+6.6	+5.6
7772.0	OI	-1.3	+1.5	-0.7	+1.4	+0.8	+1.0	+3.9	+0.5	+4.8
7774.2	OI	-0.8	+1.9	0.0	-0.5	-0.1	+1.8	+2.4	+5.4	0.0
7775.4	OI	+2.1	-0.8	+0.4	+3.0	+0.9	+5.5	+3.9	+7.0	+5.0
7780.6	FeI	-0.4	-0.9	+0.8	+2.1	+0.4	+2.5	+5.3	+6.0	+5.8
7789.0	NiI	-0.1	-0.5	-0.5	-0.4	-1.1	+1.2	+4.9	+6.1	+6.9
7797.6	NiI	-0.1	0.0	+1.3	+0.8	+0.3	+3.3	+5.4	+7.3	+5.5

CHAPTER V.

DISCUSSION AND ANALYSIS OF THE OBSERVATIONAL DATA.

1. The Term-Dependency of the Limb-Centre Shifts.

In comparing the limb effect of different lines measured on the same plates, it is important to realise that observational errors which might have been produced by small displacements of the plate holder or other apparatus between consecutive exposures, and any field effects that might have arisen through localised Doppler motions in the solar gases, would be likely to displace the wavelengths of moderately-intense Fraunhofer lines in a similar manner; hence one might expect a higher degree of consistency among the observed shifts of different lines for a given comparison than among the shifts of the same line for different comparisons*). This implies that at every spectral region it may be possible to detect small systematic

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*) This expectation was confirmed by values obtained from measurements of the control plates.

differences of behaviour across the disk with such properties as line strength ($\frac{W}{\lambda}$) and lower excitation potential (χ_L), despite the rather low degree of absolute accuracy (viz. about ± 2 mA) which characterises the values of the mean wavelength shifts quoted in Tables 15 and 16.

The influence of χ_L upon the magnitude of the mean limb-centre shifts of FeI lines measured on the Arcetri spectrograms can be inferred from an inspection of the figures quoted in Table 17, which were obtained by grouping the results for pairs of lines with similar excitation potentials at each of the four spectral regions included in this programme: the tabulated values exhibit a systematic tendency to increase with increasing χ_L, the exceptions to this rule being too few, and too small, to be significant.

The values of the relative wavelength displacements of the three pairs of lines with $\bar{\chi}_L = 3.6$ at the three disk positions closest to the edge of the Sun - where the estimated average error of about $\pm 2 \times \frac{1}{\sqrt{2}} = \pm 1.4$ mA in each tabulated figure is less than the mean observed limb-centre shifts - show a general increase with wavelength compatible with their interpretation as Doppler effects. Bearing in mind that a direct proportionality to wavelength had already been established by the earlier researches of

TABLE 17.

The Dependency of the Limb-Centre Shifts on χ_L

for Medium-Strong FeI Lines in the Visible Wavelength Range.

Limb-Centre Shift (mA)

Spectral Region	χ_L(eV)	Disk Pos λ(A)	1	2	3	4	5	6	7	8	9
Blue	3.63	4484.2 4485.7	-0.5	-0.1	-0.5	+1.0	+0.6	+1.6	+1.6	+2.5	+3.5
	0.09	4389.2 4489.7	-0.3	-1.3	-0.5	-1.1	+0.9	-1.0	-1.5	0.0	+0.4
Green	4.42	5364.9 5367.5	+1.8	-1.5	+0.9	+0.8	+1.4	-0.3	+4.3	+4.3	+6.4
	3.62	5365.4 5379.6	+1.6	-1.5	-0.8	+1.0	-1.4	0.0	+2.9	+2.9	+3.9
	0.11	5247.1 5250.2	+0.9	-1.3	-0.4	-1.1	-1.5	-3.0	-0.4	+0.1	+3.4
Orange	4.39	6056.0 6157.7	+0.4	-0.9	+3.1	+0.1	-0.8	0.0	+2.5	+2.4	+2.6
	2.19	6151.6 6173.3	-1.8	+0.8	+0.3	-1.5	+0.6	-1.0	-1.1	+1.8	-0.3
Red	3.62	6246.3 6301.5	+0.9	+1.6	-2.4	+0.9	+3.4	+1.6	+2.8	+4.8	+9.0
	2.18	6265.1 6335.3	+0.5	-0.5	-1.3	-0.1	+1.3	-1.1	+1.4	+2.5	+7.5

Walter Adams (cf. Ch.I.3) and Evershed (cf. Ch.II.3), it was consequently decided to convert the limb-centre shifts of all lines into velocity units; this procedure (which is generally adopted nowadays) permits one to base the solar limb effect of medium-strong FeI lines on different sets of plates referring to the entire wavelength range between 4400A and 7800A, thereby reducing errors due to field effects.

Since a statistical investigation of the residuals formed by subtracting the average limb-centre shifts for each spectral region from the corresponding values for individual lines summarised in Tables 15 and 16 failed to yield evidence of an appreciable correlation with line strength*), this factor was ignored in grouping the FeI lines: the four groups of lines for which the mean limb-centre velocity shifts tabulated in Table 17 were determined, are listed on the following page. Although the estimated probably error in each of these group values is of the order of ± 0.05 km/sec., differences found among corresponding velocity displacements at all nine selected disk positions along the Sun's polar diameter are systematic and appear

*) It is necessary to emphasise that this result can be considered as valid only within the limited range of line strength covered by the data under discussion.

List of Solar FeI lines used to Investigate the Term-Dependency of the Limb-Centre Wavelength Displacements.

Group a

λ(A)	χ_L(eV)
5364.9	4.43
5367.5	4.40
5383.4	4.29
6157.7	4.06
7780.6	4.45

$\bar{\chi}_L$ = 4.33

Group d

λ(A)	χ_L(eV)
4389.2	0.05
4489.7	0.12
5247.1	0.09
5250.2	0.12
6358.7	0.86

$\bar{\chi}_L$ = 0.25

Group b

λ(A)	χ_L(eV)
4484.2	3.59
4485.7	3.67
5365.4	3.56
5379.6	3.68
6246.3	3.59
6301.5	3.64
7748.3	2.94

$\bar{\chi}_L$ = 3.52

Group c

λ(A)	χ_L(eV)
4442.3	2.19
4447.7	2.21
5322.0	2.27
6137.0	2.19
6265.1	2.17
6297.8	2.21
6335.3	2.19

$\bar{\chi}_L$ = 2.20

TABLE 18.

The Observed Limb Effect of Medium-Strong FeI Lines originating from Differing Levels of Excitation.

Limb-Centre Shift (Km/sec)

Group	No. of Lines	$\cos\theta$ / χ_L (eV)	.97	.91	.80	.69	.54	.43	.27	.22	.155
a	5	4.33	±.06	-.05	+.07	+.06	+.07	+.03	+.22	+.20	+.29
b	7	3.52	+.02	.00	-.06	+.05	+.04	+.06	+.14	+.20	+.29
c	7	2.20	-.01	-.01	.00	+.02	+.05	+.02	+.04	+.12	+.28
d	5	0.25	+.02	-.07	-.03	-.06	.00	-.09	-.02	+.04	+.19

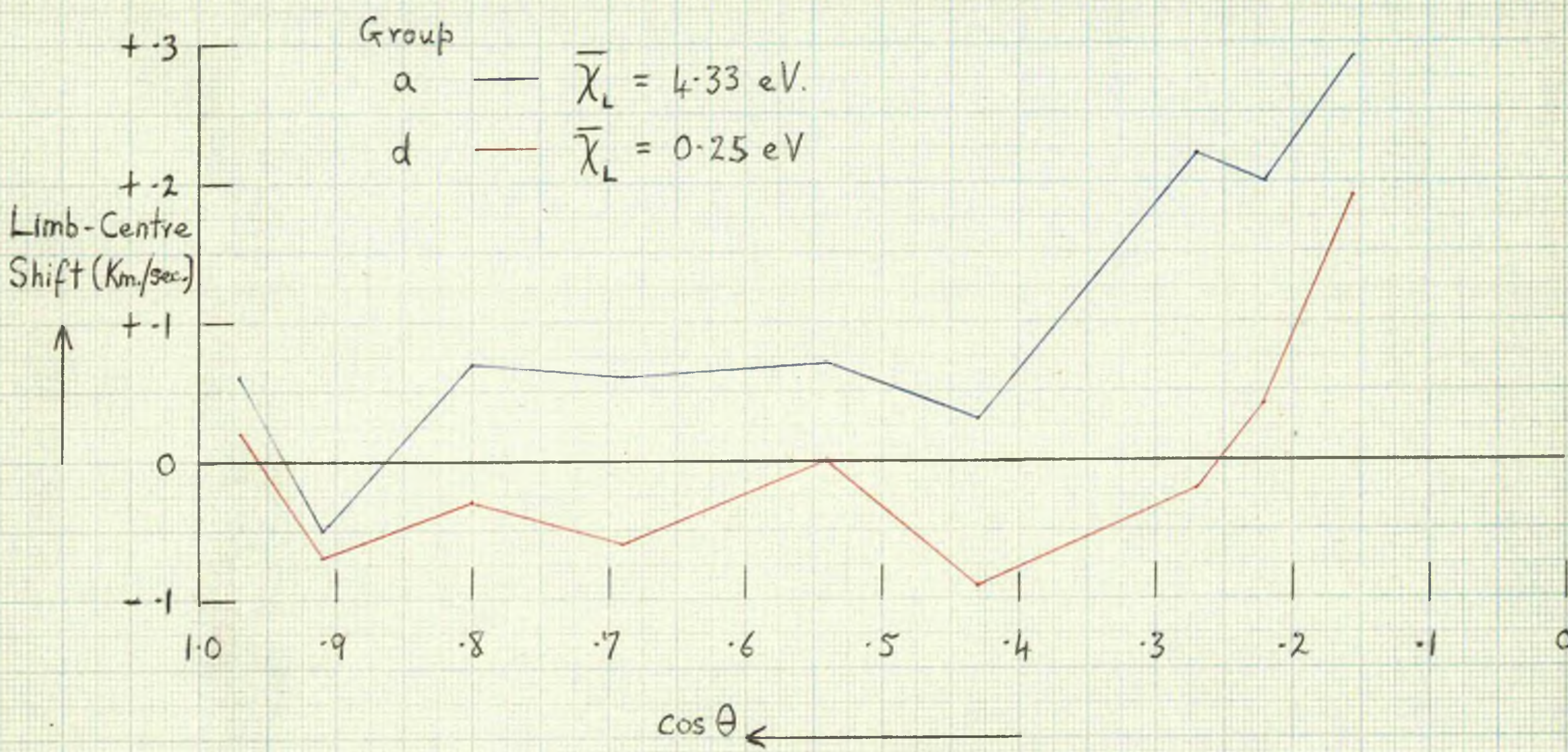

FIG.12: The Solar Limb Effect for Different Levels of Excitation.

to result from a term-dependence in the observed values. The reality of this effect is revealed by the graphs (for groups a and d) drawn in Fig.12: its discovery emphasises that the form of the solar limb effect is governed by causes that are operative over the whole range of disk positions, a fact which is vital to the interpretation of this phenomenon (cf. Ch.V.5).

The existence of a term-dependency among the values of limb-centre shifts close to the edge of the Sun's disk has actually been suggested by Luise Herzberg on the basis of observations on infrared SiI lines near 11,000A[1]; however, the extension of her programme to include the spectral regions near 8500A and 8900A[2] later failed to donfirm Herzberg's opinion. The present investigation would therefore appear to be the first to positively establish this observational feature.

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1) Herzberg, L.: Canadian Journal of Physics, 35, 766, 1957.
2) " " " " " " 38, 853, 1960.

2. Anomalous Limb-Centre Shifts in Multiplet Lines.

An interesting anomaly which Herzberg observed, but was unable to explain, was that one of the SiI lines belonging to the transition $4p\,^3D - 4d\,^3F^o$ exhibited a mean limb-centre displacement which was significantly larger than the corresponding shifts of the other two lines in this multiplet on which measurements had also been made. Since these SiI lines all appear to be exceptionally free from disturbing blends, as evidenced by the high internal consistency of the atomic energy levels derived from the solar wavelengths,[1] the systematic difference would seem to be real, despite the limited accuracy of the data.

The existence of a similar variation in the magnitude of limb-centre shifts among lines of the same multiplet is also shown by the results of our measures on six of the nine FeI lines belonging to the transition $a\,^5P - y\,^5D^o$: these lines are listed in Table 19 in order of decreasing line strength, along with the relative velocity differences corresponding to the values of the respective limb-centre wavelength shifts taken from Table 15. Now the total uncertainty inherent in the latter may amount to ± 2 mA (for an individual line), so the corresponding error involved in

1) Moore, C.E.; J.O.S.A., 43, 1014, 1953.

TABLE 19.

The Observed Limb Effect for Lines belonging to the Multiplet Transition $a^5P - y^5D^o$ ($\bar{\chi}_L$ = 2.19 eV).

Limb-Centre Shift (Km/sec)

λ(A)	Cosθ / $\frac{W}{\lambda}\times 10^6$	.97	.91	.80	.69	.54	.43	.27	.22	.155
6335.3	16.7	-.02	-.04	-.04	+.04	+.09	-.09	+.17	+.11	+.36
6265.1	13.1	+.07	-.01	-.07	-.05	+.04	-.02	-.04	+.13	+.36
6137.0	11.3	-.10	+.15	+.09	+.01	-.04	+.01	+.12	+.10	+.34
6297.8	11.0	-.01	+.10	+.02	.00	+.18	-.04	-.02	+.12	+.25
6173.3	10.9	-.18	+.05	-.02	-.10	-.01	-.01	-.05	+.07	-.02
6151.6	8.0	+.01	+.02	+.05	-.05	+.07	-.09	-.06	+.10	.00

each of the values tabulated in Table 19 may be as much as ±0.10 km/sec.; yet large as this is, it is still insufficient to account wholly for the systematic discrepancy between the mean results obtained at cos θ = .155 for the first four, and the last two, lines in this table. Of the three lines contained on the same set of plates (those centred near 6100A) for which the relative errors should be smaller, one (λ6137.0) exhibits shifts that generally tend to be higher than those found for the other two, the anomaly being particularly pronounced at the outermost limb position. An inspection of the individual plate values for the group of lines in question (cf. pp.138-9 of this thesis) reveals that the limb-centre shifts derived for the estimated position cos θ = .155 from four different plates are self-consistent, and thus emphasises the fact that the discrepancy is real.

In view of the importance of this discovery, further measurements were carried out on another six plates in the same wavelength region, and the mean results were as follows:-

λ(A)	Mean Limb-Centre Shift at cosθ=.155(based on plates 01-10) (mA)	(km/sec)
6137.0	+5.4	+.26
6173.3	+1.4	+.07
6156.6	+2.0	+.10

The figures derived from the ten plates 01-10 (cf.Table 8, p.104) are seen to be in better agreement; yet it is unlikely that the anomaly has arisen entirely through selection errors, for on the assumption that the errors in the individual plate values follow a normal distribution, the estimated error of the velocity shifts quoted above should be reduced to $\frac{\sqrt{4}}{\sqrt{10}} \times \pm 0.10 = \pm 0.06$ km/sec., which is only a third of the observed difference. Although one might attribute the observed discrepancy to the presence of undiscovered blends in the immediate neighbourhood of λ6173.3 and λ6151.6, a more plausible explanation could be that it is related to line strength; for the mean velocity shifts at cos θ = .155 quoted in Table 19 indicate a dependency upon $\frac{W}{\lambda}$, in the sense that the weaker lines are associated with the smallest limb-centre displacements.

3. Features of the Solar Limb Effect Observations near 7770A.

An effect revealed by the infrared observations near 7770A which is not detectable in the data collected for the visible spectral range is the small decrease in the majority of the values of the mean limb-centre shifts at the outermost disk position (cos θ = .155) as compared with those at cos θ = .22 (cf.Table 16). Since the seeing conditions at Göttingen were inferior to those at Arcetri (cf.Atmospheric conditions in Table 11 and Tables 6-9) the question arises as to whether the observed feature may be associated with the scattering of light in the Earth's atmosphere. Until recently, it was generally taken for granted - following the negative result of an investigation by Brück[1)] concerning the effect of the atmospheric scintillation on the half-widths and residual intensities of selected Fraunhofer lines near 4100A and 6200A - that this phenomenon would exert no appreciable influence on the wavelengths of solar spectral lines; however, the limb effect data recently collected at Oxford with the help of the newly-installed high-resolution equipment have indicated a systematic difference between the limb-centre shifts derived for the East and West equatorial radii (in the range $.20 \geq \cos\theta \geq .00$) which increases quite rapidly from approximately 0.04 km/sec. at $\cos\theta = .22$ up to 0.18 km/sec. at the edge of the solar disk.[2)]

From the above figures one might infer that the corresponding corrections to be applied to observations made along the polar diameter vary from 0.02 km/sec. to 0.09 km/sec. over the range of θ in question. Thus the limb-centre shifts observed near $\cos\theta = .155$ may require to be increased

1) Brück, H.; Zeit.f.Ap., 1, 58, 1930.
2) Adam, M.G.; M.N., 119, 460, 1959.

by about +0.05 km/sec.[*]; but for the other eight disk positions at which spectral comparisons were obtained, the scattered light correction ought to be negligible. Strictly, this estimate applies to moderate seeing conditions and is restricted to the wavelength region near 6300A to which the Oxford observations refer; consequently, in view of the uncertainty incurred by adopting such a correction for a different spectral region, and the difficulty of defining the final limb position accurately, no adjustment to any of the values obtained from the presant investigation has been made to compensate for the effect of scattered light.

In accordance with the result deduced from our discussion of the Arcetri material, the higher excitation potential NiI line (λ7796.6) exhibits systematically larger wavelength shifts than the other selected NiI line (λ7789.0). (cf. Table 16). The sole exception to this general trend is found at the final limb position, but the discrepancy is too small (viz. 1.4 mA) to be considered as significant. The wavelength shifts measured for the OI triplet - though not very consistent - showed no systematic variation in the form of the centre-limb increase, in accordance with expectation since all three lines originate from the same

*) This velocity shift corresponds to a wavelength displacement of 1.3 mA at 7770A.

lower level of excitation. There is no suggestion either of a term-dependency among the three infrared FeI lines, but the range in χ_L may be too narrow, and the observational errors too large, for a small effect of this nature to be evident. Nevertheless, there does appear to be a slight tendency for the limb-centre shifts of the line λ7751.2 to be smaller than the corresponding values for the other two FeI lines (cf.Table 16), but this behaviour may arise through the former being very weak in comparison with the latter (cf. previous section).

The mean shifts for the three elements comprising the infrared data (FeI, NiI, OI) - found by averaging the observed line shifts given in Table 16 - have been quoted (in velocity units) in Table 20 and are represented graphically in Fig.13. The limb effect for FeI and NiI is seen to be effectively the same, the only notable discrepancy being at cos θ = .27; but the latter is entirely due to the influence of the low value of the limb-centre shift obtained for the weak FeI line λ7751.2, the mean displacement for the other two FeI lines alone being exactly the same as that found for the two NiI lines. An equally close agreement among the limb-centre shifts of FeI and NiI lines near 8500A and 8900A has also been obtained by Herzberg[1], which adds weight to

1) Herzberg, L.; Canadian Journal of Physics, 38, 853, 1960.

the generality of our result as far as these two elements are concerned. On the other hand, it appears that the centre-limb increase of the three OI wavelengths is rather less pronounced, a feature that may be associated with the fact that they are weak (cf. previous section) and originate at very much deeper levels inside the solar atmosphere than the FeI and NiI lines included in our programme.

TABLE 20.

The Observed Limb Effect near 7770A for Different Elements.

Limb-Centre Shift (Km/sec)

El.	No. of Lines	$\cos\theta$ / χ_L(eV)	.97	.91	.80	.69	.54	.43	.27	.22	.155
FeI	3	4.12	-.03	+.02	+.02	+.01	-.01	+.09	+.15	+.25	+.24
NiI	2	2.91	.00	-.01	+.02	+.01	-.02	+.09	+.20	+.26	+.24
OI	3	9.11	.00	+.03	.00	+.05	+.02	+.11	+.13	+.17	+.12

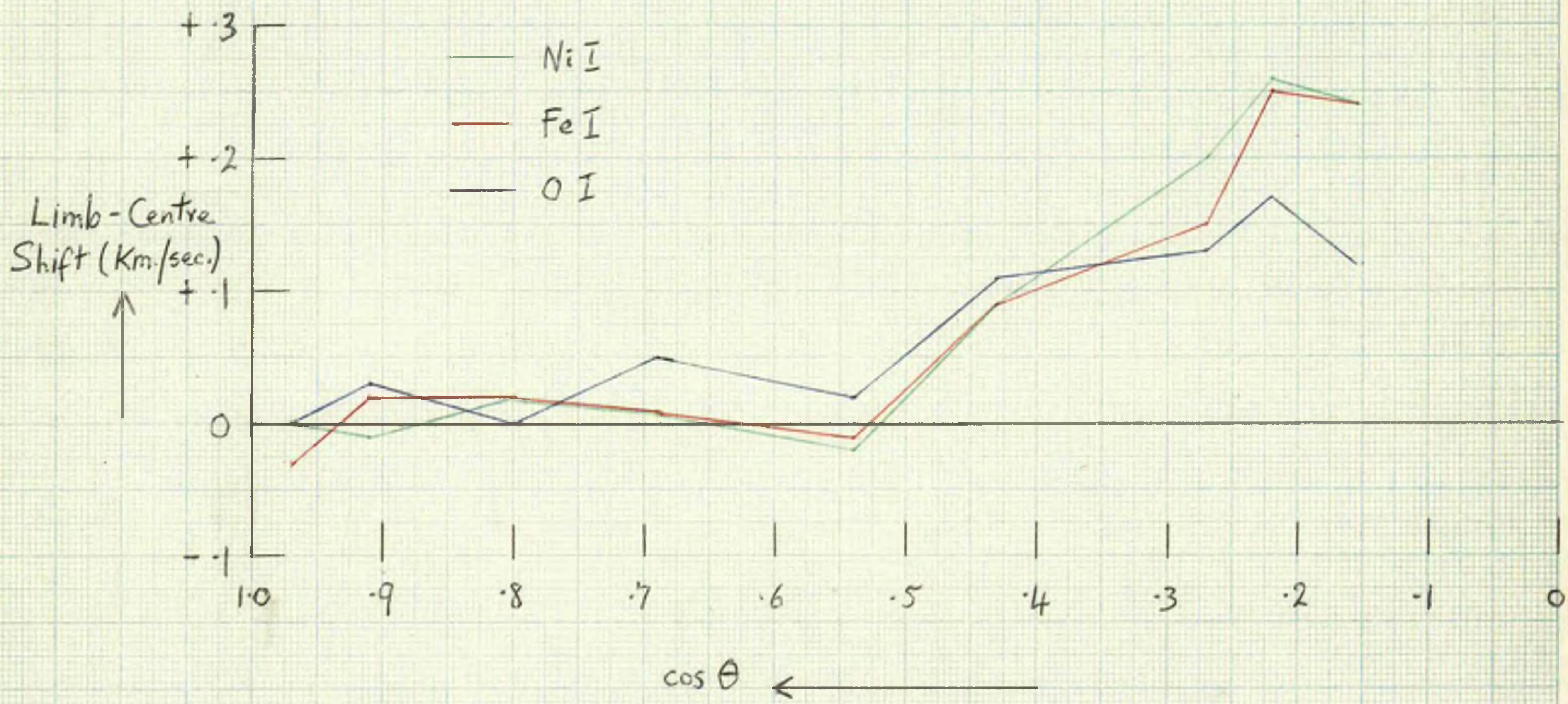

FIG.13: The Solar Limb Effect at 7770A for Different Elements.

4. Comparison of the Present Observations with Independent Limb Effect Data.

In the foregoing discussion of the Arcetri and Göttingen observations we have established the existence of a dependency of the limb-centre shifts on χ_L, and have found some indication that the former may also be weakly correlated to $\frac{W}{\lambda}$ for the limited ranges in these physical parameters covered by the type of solar lines included in the present investigation. In addition, the results for lines similar with regard to the first of the above-mentioned properties were combined on the basis of the assumption that the wavelength variation was completely taken into account by expressing the relative shifts in velocity units. Now Herzberg (1960) has explicitly stated that the observed mean velocity shifts of (moderately-intense) FeI lines near 8500A and 8900A are practically identical to those obtained by Adam (1948) for neutral metal lines near 6100A; while from a comparison of the latter with the limb effect data obtained by the Potsdam observers (1930) which refer to the spectral region near 4400A (cf.Ch.II.4), Adam concluded that "there is no significant change in the form of the relation with time, or with wave-length region in the solar spectrum". (cf. M.N., 108, 458, 1948). Recently, the limb effect measurements by

Schröter[1] on 9 FeI lines in the wavelength regions 6287-6337A and 6471-6529A, and those by Adam[2] and Higgs[3] on three moderately strong FeI lines near 6300A, have also been found to yield mutually consistent results, ~~within the limits of error set by field effects arising from the turbulent motions of the solar gases.~~ Thus - to within the limits of error set by field effects arising from the turbulent motions of the solar gases - the generality of the form of the limb effect is confirmed for the visible and near infrared solar spectrum by the agreement among the independent results obtained by observers working with different equipment and under different climatic conditions.

In view of this fact, the solar limb effect for medium-strong FeI lines derived on the basis of the present investigation has been obtained by averaging the values of the limb-centre differences for all 31 lines (belonging to this element) which were selected for measurement. This procedure should reduce the average error in the final figures to ≤0.05 km/sec., or to less than half of that likely to be involved in the mean values of velocity shifts of medium-strong lines contained upon the same set of plates, for which the field effects are the same. A comparison between the results thus obtained and the independent data mentioned above (e.g. see Fig.15)

1) Schröter, E.H.; Monatsber.d.Dt.Akad.d.Wiss., Berlin, 1, 738, 1959 (cf.Fig.2)

2) Adam, M.G.; M.N., 119, 460, 1959.

3) Higgs, L.A.; M.N., 121, 421, 1960.

~~which it would appear~~ indicates that the former are significantly lower than the values of the limb-centre shifts determined by the other observers. However, a glance at Fig.12 serves to remind us that our final limb effect curve must be depressed - particularly at cos θ = .43 - by the results for lines of low excitation potential: a set of mean values based upon groups a and b (Table 18), with $\bar{\chi}_L$ = 3.86, are found to be ~ 0.05 km/sec. larger than the plotted figures in the region of the disk from cos θ = .40 - .00, and this effect is capable of accounting for about one-third of the discrepancy.

The possibility that observational errors incurred by the writer account for the remaining part tends to be ruled out by the fact that the form of the increase close to the limb found from the present investigation is in excellent agreement with the results obtained by Hart[1)] from velocity measurements, which she regarded as being "not in disagreement" with limb effect data collected by Adam. Although Hart was originally of the opinion that the larger limb-centre shifts and the steeper rise to the limb shown by Adam's interferometric observations (1948) was merely a consequence of the low telescopic resolution necessarily

1) Hart, A.B.: M.N., 114, 17, 1954.

employed in the letter investigation, the later Oxford researches (1959, 1960) yield evidence of an even greater discrepancy with Hart's velocity measurements (and consequently also with the presently derived data) which can no longer be attributed to this cause. Further work by Hart[1] has indicated that root-mean-square velocity fluctuations of $\sim\pm0.15$ km/sec. can be expected at a given point on the solar disk due to large scale gaseous motions in the equatorial region of the Sun's photosphere, while Adam is of the opinion that displacements of between 0.05 km/sec. and 0.10 km/sec might arise from uncertainties produced by field effects (cf.M.N.,119,473,1959), and these seem large enough to account for the major part of the systematic difference between our results and those found by other observers.

Additional complications liable to affect the consistency of the limb effect data have recently been brought to light by Plaskett[2], who detected an asymmetry in the solar rotation and the presence of meridional currents on the solar surface with velocities comparable with those of the excess equatorial acceleration. The previous neglect of these newly-discovered features could have introduced appreciable errors in the corrections generally applied to

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1) Hart, A.B.; M.N., 116, 38, 1956.
2) Plaskett, H.H.; M.N., 119, 197, 1959.

the observed relative displacements in order to compensate for the solar rotation, which would have been reflected in the values derived for the solar limb effect. Meridional currents along the polar diameter - where no correction for solar rotation need be applied - flowing towards higher solar latitudes may well have contributed towards the systematic depression shown by the present limb effect curve. The effect of scattered light may have had a slight influence on the limb-centre shifts observed at $\cos\theta = .155$ (cf.Ch.V.3), but if Adam's analysis (1959) is valid, it should have no observable effect at the other eight disk positions that lie within the range $1.00 \geq \cos\theta \geq .20$. Other factors involved may be the presence of undiscovered blends in the immediate neighbourhood of some of the solar lines selected for measurement, and the presence of unexplained asymmetries in certain line profiles observed near to the Sun's limb (cf.Higgs; M.N., 121, 434, 1960).

5. A Physical Interpretation of the Solar Red Shifts for Moderately-Intense Lines.

The only physical theory so far proposed in an attempt to explain the solar red shift observations is the relativity-radial current hypothesis introduced by St. John (1928) and revised later by Schröter (1957). The concepts underlying Schröter's treatment of the data which existed at that time, and the difficulties confronting his interpretation, have already been discussed in Ch.II.5. Following Schröter, we assume (provisionally) that the centre-limb variation in the absolute wavelength of a moderately-intense Fraunhofer line is completely determined by two superimposed physical effects: viz. the predicted gravitational red shift (= 0.636 km/sec.), for which there now appears to be some observational support (cf. Ch.I.7), and systematic Doppler effects associated with the temperature fluctuations known to persist in the solar photosphere. Accordingly, we represent the observed value (in km/sec.) of the wavelength difference between a solar and a terrestrially-excited spectral line ($\Delta\lambda_\theta$) by an equation of the form:-

$$\Delta\lambda_\theta = 0.636 - v_\theta \cos\theta$$

$$\text{or} \quad \Delta\lambda_\theta = 0.636 - v_o\beta_\theta \cos\theta \qquad (1)$$

$$\text{where} \quad \beta_\theta = \frac{v_\theta}{v_o} \qquad (2)$$

The quantity v denotes the effective granular displacement resulting from the superposition of the two mean intensity profiles derived for the granular and intergranular columns respectively by integrating the displaced elementary contours over all optical depths in which the spectral line is being formed; thus it is intimately related to the shape of the observed intensity profile and the depth-dependency of the postulated radial currents. In virtue of the fact that one sees into progressively shallower layers of the Sun's atmosphere as one observes closer to the edge of its disk, v can generally be regarded as a continuous function of the angle of emergence θ; its quantitative determination on the basis of a schematised inhomogene model atmosphere involves detailed numerical calculations best carried out on an electronic computer such as the IBM 650. The calculations are simplified in the case of moderately-intense lines (with which we are presently concerned) since for these the net wavelength displacement resulting from the superposition of line profiles for the granular and intergranular columns should not be appreciably affected by damping. Furthermore, Schröter has demonstrated that over all but the extreme edge of the disk (from $\cos \theta = .01 - .00$) where observational difficulties are likely to outweigh those introduced by the numerical method, the form of the centre-limb variation of v_θ - hence of β_θ - for such lines is wholly determined by the continuum contrast.

Thus, in general, no serious error should be incurred if - when dealing with moderately-intense lines - one neglects the centre-limb variations in the residual intensity and depth-dependency of the velocity field, and represents β_θ by the approximate theoretical relation (cf. Schröter, Zeit.f.Ap., 41, 177, 1957):-

$$\beta_\theta = \frac{H_\theta - 1}{H_\theta + 1} \bigg/ \frac{H_o - 1}{H_o + 1} \quad \text{where } H_\theta = \frac{I^G(\theta)}{I^{IG}(\theta)} \qquad (3)$$

Hence β_θ can be calculated from this equation when the values of the continuum contrast at the disk centre ($= H_o - 1$) and at other disk positions ($= H_\theta - 1$) are known. At present there is some difference of opinion concerning the validity of the methods of reduction normally employed for the purpose of deriving this quantity from photographs of the solar granulation, and no definite figure can yet be quoted; however (as has already been remarked in Ch.II.4), most observers favour a value of the intensity ratio $H_\theta > 1.10$, or a contrast $(H_\theta - 1) > 10\%$. The observational difficulty involved in making a reliable assessment of the continuum contrast and in attempting to provide a quantitative estimate of its dependency on wavelength and disk position, is undoubtedly a major factor limiting the significance that can be placed upon all presently existing inhomogene models.

In the absence of such precise information, Schröter has adopted ($H_0 - 1$) = 45% and used the resulting temperature stratifications of the granular and intergranular streams to deduce the values of $I^G(0,\theta)$ and $I^{IG}(0,\theta)$ respectively from the equation of transfer for a plane-parallel atmosphere in radiative equilibrium; thus by substituting $H_0 = 1.45$ and the calculated values of $H_\theta = \frac{I^G(0,\theta)}{I^{IG}(0,\theta)}$ into equation (3), he finally obtained the graph of $\beta_\theta \, v \cos\theta$ which we have reproduced in Fig.14 (cf.Zeit.f. Ap., 41, 1957: Abb. 8, p.178). Strictly, the derived relation refers to the spectral region close to 4100A, but Schröter's Two-Stream Model predicts a very weak wavelength dependency of the continuum contrast (in accordance with granulation observations) so it appears that the effect of the latter can be neglected.

Having now established the basis for the physical interpretation of the observed absolute solar wavelength displacements, we are now in a position to analyse the results of the existing solar limb effect observations. Defining the limb-centre velocity displacements by

$$\Lambda(\theta) = \Delta\lambda_\theta - \Delta\lambda_0 \qquad (4)$$

we find from (1) that

FIG.14: The Predicted Centre-Limb Variation of the Effective Granular Velocity Displacement of a Medium-Strong FeI Line due to the Effect of Continuum Contrast only.
(cf. Schroter, Zeits.f.Ap., 41, 1957; Abb.8).

$$\Lambda(\theta) = v_o(1 - \beta_\theta \cos \theta) \qquad (5)$$

Since one observes $\Lambda(\theta)$, and can interpolate the appropriate values of β_θ from the graph in Fig.14, the most probable value of v_o for each set of limb effect data can easily be obtained from the least-squares solution of the conditional equation (5). The results of such analyses are tabulated in Table 21 together with the values of v_o found from the statistical relations between the residual wavelength shifts*) and line strength derived independently by Adam[1] and by Schröter[2] on the basis of their interferometric observations of the centre-arc shifts: the corresponding values found from our own statistical grouping of St. John's centre-arc shifts (cf.Ch.II,Fig.2 or Table 3), which are not quoted in Table 21, are in closer agreement with Adam's results than with Schröter's.

A comparison of these latter estimates with the values of v_o found from our analysis of the limb effect measures shows them to be compatible only in the case of the present investigation; for the other five sets of data analysed, the predicted velocities are too high. More specifically,

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*) i.e. those which remain after the relativity effect has been subtracted from the observed centre-arc displacements (denoted by δ in Ch.II.1)

1) Adam, M.G.; M.N., 118, 1958: Fig.1, p.111.

2) Schröter, E.H.;Monatsber.d.Dt.Akad.d.Wiss.,Berlin,1,1959: Abb.1, p.741.

since from the provisional equation (1)

$$\Delta\lambda_o = 0.636 - v_o, \quad (6)$$

our results imply that if Schröter's interpretation were correct, the centre-arc shifts should be considerably smaller than those which are actually observed, the amount of the discrepancy being of the order of 0.30 km/sec., or nearly half the relativity effect itself.

The high quality of the work carried out at Oxford and Potsdam would appear to preclude the possibility that the excess value of the net velocity displacement derived from (6) reflects the presence of observational errors in the centre-arc shifts, while the accuracy with which equation (5) represents the limb effect data (cf. Fig. 15) would seem to guarantee the validity of Schröter's interpretation as applied to this phenomenon. The small systematic deviations from the predicted form of the centre-limb wavelength variation which do occur - in particular, the tendency shown by the limb-centre differences to assume negative values in the range $1.0 \geq \cos\theta \geq 0.8$ (cf. Schröter, 1959) - can always be attributed to errors in β_θ resulting from the wrong choice of granule contrast and the associated inaccuracies in the adopted values of the temperature fluctuations; but although the latter may disguise any variation in the amount of the discrepancy across the disk, they can

TABLE 21.

Comparison of Velocity Shifts derived from Relative and Absolute Red Shift Measurements of Moderately-Intense FeI Lines.

Limb Effect Data	Mean Wave-length $\bar{\lambda}$(A)	No of Lines	$\frac{\bar{W}}{\bar{\lambda}}\times10^6$	$\bar{\chi}_L$(eV)	v_o(km/sec.)		
					from analy-sis	Adam 1958	Schröter 1959
Forbes (1961)	5770	31	17.7	2.79	0.23	0.15	0.24
Higgs (1960)	6300	3	14.0	3.17	0.55	0.19	0.29
Adam (1959)	6300	3	14.0	3.17	0.50	0.19	0.29
* Schröter (1959)	6400	9	-	-	0.42	-	-
Adam (1948)	6130	9	12.7	3.21	0.43	0.20	0.30
Freundlich et al. (1930)	4430	9	25.7	0.07	0.49	0.11	0.18

* Full details of this work have yet to be published; in the discussion of his preliminary results, Schröter did not mention the physical properties of the lines to which his measurements refer.

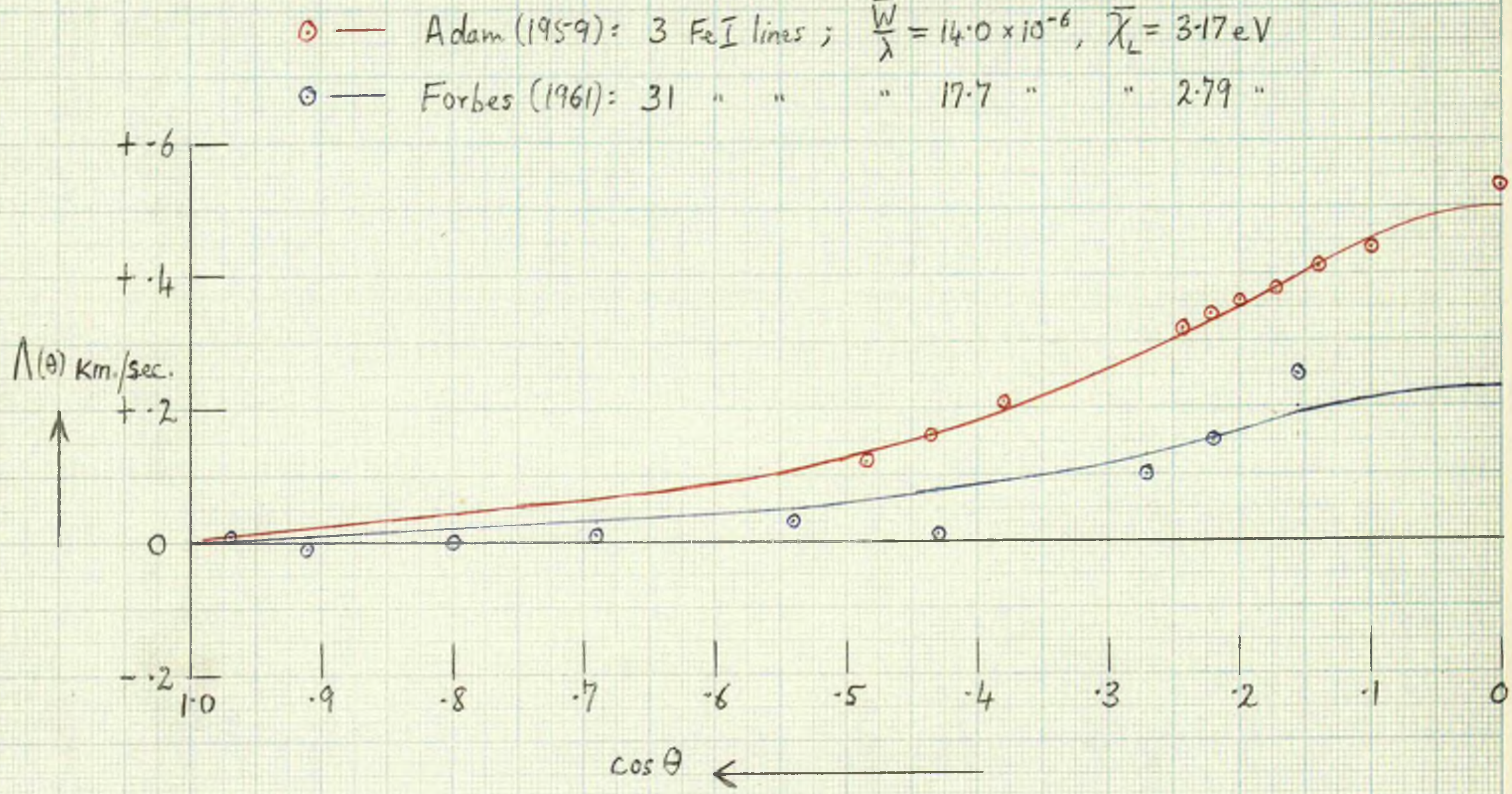

FIG.15: Representation of Limb Effect Data for Medium-Strong FeI Lines on the basis of Schroter's Two-Stream Model (1957).

certainly never account for the whole of its value.

Thus one seems left with no alternative but to postulate the existence of a third physical effect - besides the relativity and Doppler effects - producing a constant wavelength displacement of moderately-intense solar spectral lines at all points along a given radius of the solar disk. Consequently, the basic equation (1) requires to be amended to:

$$\Delta\lambda_\theta = 0.636 + \delta_0 - v_0\beta_\theta \cos\theta \qquad (7)$$

where δ_0 denotes the shift (in km/sec.) whose physical nature has yet to be established.

The precise value of δ_0 is dependent on the choice of atmospheric model, which at the present time is recognised as a matter of some uncertainty: yet its characteristic feature is that it represents a wavelength displacement to the <u>red</u> in the mean positions of solar absorption lines with respect to their (emission) counterparts excited in a terrestrial light source. A recent experiment on nuclear resonance to which we have already referred in Ch.I.7 (cf. Cranshaw et al.; Phys.Rev.Lett., <u>4</u>, 163, 1960) yields confirmation that the gravitational red shift is actually within 5% of the value predicted by Einstein, thus on the grounds of this information one would be unjustified in supposing that δ_0 is involved in the time-dilution factor, although from a phenomenological standpoint such an interpretation would remove the basic difficulty which confronts this problem: viz. that of accounting for the observed excess red shift at the limb. In the absence of any proof to the contrary, δ_0 may be interpreted on the basis of Spitzer's theory (cf.Ch.II.5) as the combined effect of collisional shifts in tending to displace laboratory lines farther to the violet, and/or solar lines farther to the red of their predicted wavelengths, but this alternative still requires positive confirmation before one is able to assess its validity. Until more information becomes available, we prefer to avoid further speculation concerning the physical nature of δ_0; at present, we merely insist that the existence of such a quantity must be admitted on empirical grounds.

6. The Relation between the Physical and Empirical Red Shift Formulae.

In order to establish the connection between the physical formulae derived in the previous section and Freundlich's empirical red shift formula in its amended form (cf.Ch.II.6), one might first express the latter in velocity units as:

$$\Delta\lambda_\theta = c.CI_\lambda(\theta)\sec\theta \qquad (0 \le \theta \le 80^\circ) \quad (8)$$

where c denotes the velocity of light ($= 3 \times 10^5$ km/sec.). Then, using definition (4), one obtains:

$$\Lambda(\theta) = c.CI_\lambda(0)(\frac{I(\theta)}{I(0)}\sec\theta - 1)$$

This equation may be written otherwise as:-

$$\Lambda(\theta) = -c.CI_\lambda(0)(1 - \varphi(\theta)\sec\theta) \qquad (9)$$

where $\varphi = \frac{I_\lambda(\theta)}{I_\lambda(0)}$ denotes the limb-darkening function for the continuous radiation, and compared with (5) to obtain the pair of relations:

$$|C\ I_\lambda(0)| = \frac{v_\Omega}{c} = (\frac{\Delta\lambda_\Omega}{\lambda})_{Doppler} \qquad (10a)$$

$$\text{and} \quad \varphi_\lambda(\theta)\sec\theta = \beta_\theta\cos\theta \qquad (10b)$$

The connection between $\varphi_\lambda(\theta)$ - which we now regard as being the mean limb darkening resulting from the combination of the respective limb darkening for the granular and intergranular streams - and the quantity β_θ arises from the fact that both are functions of the normalised granule intensity ratio $\frac{H_\theta}{H_0}$; for, by definition:

$$H_\theta = \frac{I^G(\theta)}{I^{IG}(\theta)} = \frac{I^G(0).\varphi^G(\theta)}{I^{IG}(0).\varphi^{IG}(\theta)} = H_o \frac{\varphi^G(\theta)}{\varphi^{IG}(\theta)}$$

$$\text{or} \quad \frac{H_\theta}{H_o} = \frac{\varphi^G}{\varphi^{IG}} \qquad (11)$$

Hence, from (3) and (11),

$$\beta_\theta = \beta(\frac{H_\theta}{H_o}) = \beta(\frac{\varphi^G}{\varphi^{IG}}) = \beta(\varphi)$$

The relation (10b) holds only so long as β_θ is determined by the effect of the continuum contrast; this implies that the validity of the empirical formula is confined to lines of moderate intensity, whereas the quantity β_θ has a more general significance.

Since we have interpreted the value of v_o given by (5) as being the contribution to the observed solar red shifts of the Doppler currents alone, the direct correspondence between the observed quantities $I_\lambda(0)$ and v_o evidenced in (10a) reflects a basic proportionality between the net effect of granular shifts and the value of the source-function at a given optical depth. Under the physical conditions known to be prevailing in the solar atmosphere the source-function can be identified with the Planck function and represented to a high degree of approximation by an equation of the form[1]:

1) Milne, E.A.; Hdbuch.d.Physik, III, (Part 1), 113, 1930.

$$B(\tau) = a_1 + a_2\tau,$$

and so one might tentatively infer that the effective velocity shifts produced by temperature fluctuations between the granular and intergranular columns must also exhibit a linear variation with optical depth. Indeed, calculations by the writer based on this assumption, combined with a theoretical expression for the observed line-of-sight velocity derived by Mitra[1] that takes no account of additional line shifts associated with the physical properties of Fraunhofer line formation, have shown that the former yields an excellent representation of all existing limb effect data for moderately-intense lines, for which it is deemed valid.

Further information concerning the velocity distribution with depth inside the solar atmosphere is provided by Vitense's theory of the hydrogen convection zone[2], which predicts the existence of large temperature fluctuations of as much as 1000°K (or $300 - 400^\circ$K on the average) at a given <u>optical</u> depth - associated with large streaming velocities of 2 - 3 km/sec. in the granulation elements - persisting up to $\tau_{\lambda 5010} = 0.3$ where they vanish due to

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1) Mitra, K.K.; Observatory, <u>59</u>, 160, 1936.
2) Vitense, E.; Zeit.f.Ap., <u>32</u>, 135, 1953.

the dissipation of the granule energy to the surrounding medium. Independent observational facts supporting the disappearance of the temperature differences near this limit have been summarised by Professor de Jager in his recent memoir[1].

7. Analysis of the Observed Wavelength Displacements of Intense Solar Absorption Lines.

Owing to the large effective optical depth of the cores of strong (easily-excited) lines, the deeper layers - in which the granular Doppler shifts are largest - make a negligible contribution to the absorption, and become effective only in the damping portion of the line profiles; consequently, in visual line-shift measurements the net effect is governed by the velocities corresponding to the layers in which the cores are being formed (cf. Schröter, Zeit.f.Ap., 41, 175, 1957). Thus the centre-arc shifts and limb effect of lines whose cores are formed at levels where the temperature fluctuations due to granulation are small (i.e. for $\tau_{\lambda 5010} \sim 0.3$) should be adequately represented by putting $v_o = 0$ in equations (7) and (5) respectively: this procedure yields

1) de Jager, C.: Hdbuch.d.Physik.,LII, 88-9, 1959.

$$\left.\begin{array}{l}\text{for the centre-arc shifts: } \Delta\lambda_o = 0.636 + \delta_o > \Delta\lambda_{rel} \\ \text{and for the limb-centre shifts: } \Lambda(\theta) = 0\end{array}\right\}$$

These conditions are both satisfied by Evershed's observations on two strong AlI lines in the ultra-violet region of the spectrum, and the Na-D lines (cf.page 52). Recent measurements by Schröter have also failed to yield evidence of a systematic centre-limb variation in the wavelengths of the letter pair of lines[1].

On the other hand, Evershed's observations of the H and K lines in the "reversing layer" and in prominences (cf.Ch.II.3), and Herzberg's determination of the limb-centre displacements of two very intense infrared CaII lines[2], provide adquate proof for the existence of a negative limb effect: viz. $\Lambda(\theta) < 0$. By equation (5), this condition implies that $v_o < 0$, which corresponds to a net velocity of recession of the chromospheric gases. This behaviour was noted by St. John, who was unable to offer a satisfactory explanation of it (cf.page 37); however, it can now be understood physically on the basis of an inhomogene model atmosphere, according to which the effective velocity v_o is regarded as the resultant velocity

1) Schröter, E.H.; Monatsber.d.Dt.Akad.d.Wiss.(Berlin), **1**, 743, 1959.
2) Herzberg, L.; Can.J.Phys., **38**, 863, 1960.

arising from the higher (but counteracting) velocities of the granular streams - the latter entering with different weighting factors which depend upon the intensity ratio (or temperature fluctuations) between the granulum and intergranulum at each optical depth. Direct observational evidence that the temperature inhomogeneities disappear and then re-establish themselves in the chromosphere and upper photospheric layers is provided by the spectrograms obtained with the vacuum spectrograph at the McMath-Hulburt Observatory[1], which clearly reveal asymmetries in the weaker lines which are pronounced at the centre of the disk and vanish towards the limb, whereas the higher-level lines show a greater "wriggling" which becomes more pronounced towards the limb. The fact that these zig-zag structures are most evident in the strong chromospheric lines can be interpreted as due to the larger size of the granules in the outer layers.

The centre-arc shifts of very intense (chromospheric) lines are obtained by putting $v_0 < 0$ in equation (7); thus at the centre of the disk:

$$\Delta\lambda_0 = 0.636 + \delta_0 + |v_0| \qquad (v_0 < 0) \qquad (12)$$

To separate the influence of $|v_0|$ from that of the

1) McMath, R., Mohler, O.C., Pierce, A.K. and Goldberg, L.: Ap.J., 124, 1, 1956.

unexplained δ_0 one also needs to know the value of the limb-arc displacement close to the edge of the disk, where the Doppler effect of radial currents will be negligible; hence at the edge of the disk:

$$\Delta\lambda_\theta = 0.636 + \delta_0 \quad (\text{for all } v_0) \qquad (13)$$

From his extensive data on the shifts of the H and K lines in prominence spectra, Evershed concluded that the mean value of $\Delta\lambda_\theta$ just outside the limb is between +5 mA and +6.5 mA in excess of the relativity shift (cf.Ch.II.3): these figures correspond to velocity displacements of +0.38 km/sec. and +0.49 km/sec. respectively, and by (13) they must define the limiting values of the quantity δ_0. The lower value is in good agreement with that anticipated by our analysis of the limb effect data for moderately intense lines (viz. $\delta_0 \sim +0.30$ km/sec.), while the average value of

$$\delta_0 = \frac{0.38 + 0.49}{2} = 0.43_5 \text{ km/sec.}$$

is appreciably higher than the formerly derived estimate. The mean centre-arc shift of the H and K lines was found by Evershed, and confirmed independently by Jackson, to be +21 mA (cf.Ch.II.3). This corresponds to

$$\Delta\lambda_0 = +1.59 \text{ km/sec.};$$

hence from (12) we deduce

$$|v_o| = \Delta\lambda_o - 0.636 - \delta_o = 1.59 - 0.636 - 0.435 = 0.52 \text{ km/sec.}$$

Recent interferometric measurements by Nichols[1] have yielded mean centre-arc shifts of +9.2 mA and +18.5 mA for the H and K lines respectively, which are considerably lower than the corresponding values of +19 mA and +23 mA given by Jackson (1931); in view of this, no great reliance can be attached to the quantitative values derived above.

The results quoted by Herzberg (1960) for the two very intense (low excitation) CaII lines λ8498.1 and λ8542.1 are as follows:-

At the centre: $\Delta\lambda_o$ = +0.050A or +1.76 km/sec.

At cosθ = .19: $\Delta\lambda_\theta$ = +0.031A or +1.09 km/sec.

Substituting the observed value of $\Delta\lambda_\theta$ into (13), one immediately finds δ_o = +0.45 km/sec., which is seen to be in excellent agreement with the average value obtained from measurements of the H and K lines in prominence spectra. Subtracting (13) from (12) yields

$$|v_o| = \Delta\lambda_o - \Delta\lambda_\theta = 1.76 - 1.09 = 0.67 \text{ km/sec.}$$

This estimate is independent of the errors likely to be involved in the solar and laboratory wavelengths, its accuracy depending only upon the reliability of Herzberg's

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1) Proc.of Obs.(Oxford): M.N., 118, 337, 1958.

relative measurements and on the assumption that the Doppler effects exert no appreciable influence upon the wavelengths of the infrared CaII lines at $\cos\theta = .19$. Despite the great difficulty involved in making visual micrometer settings on the centre of very diffuse lines, the results for the two pairs of CaII lines can be considered as additional proof for the existence of a significant red shift which cannot be accounted for on the basis of the relativity-radial current interpretation.

8. Conclusion.

The algebraic decrease in v_o from a positive to a negative value appears to be related fundamentally to the mean optical depth to which the line-shift measurements refer[†)]. The resulting increase of the observed (absolute) red shifts with increasing height in the solar atmosphere is reflected by:

(i) the strong intensity dependence of the centre-arc shifts, which gives rise to over-relativistic values for the more intense Fraunhofer lines (cf. Ch.II.1);

(ii) the progressive change of the limb-centre differences from positive to negative values (at a given disk position) with increasing line

†) In the case of visual micrometer determinations this corresponds to the effective optical depth in the line core.

intensity (cf.Ch.V.7 and the results of the pioneer investigation by Evershed and Royds in Ch.I.3, pp.13 and 14); and by

(iii) the tendency shown by the limb-centre shifts of moderately-intense lines to increase with the properties χ_L and $\frac{W}{\lambda}$ (cf.Ch.V, 1 and 2).

The importance of the line strength ($\frac{W}{\lambda}$) - more so than the level of excitation (χ_L) - in determining the amount of these relative wavelength variations may be judged from the fact that Herzberg's observations on another pair of CaII lines near 8900A for which χ_L = 7.0 eV but whose intensity and general appearance correspond to that of an average line in the solar spectrum, yield values for the limb-centre displacement near the limb (at cos θ = .19) only half as large as, and in the opposite direction to, those of the pair of very intense but low excitation (χ_L = 1.7 eV) lines mentioned in the above discussion.

The present investigation has served to corroborate the belief that the observed solar red shifts must result from a superposition of at least three independent effects:viz.

(1) The predicted gravitational red shift (= 0.636 km/sec.).

(2) The granular Doppler shifts associated with the temperature inhomogeneities which give rise to the phenomenon

of solar granulation.

(3) A red shift of unestablished origin, that may be supposed constant across the solar disk: it is this effect which causes the values of the limb-arc shifts to exceed that of the relativity effect.

The results described in this last chapter suggest that the value of this third effect is roughly ½ of the relativity shift, but more observational data covering a wide range of line intensity, excitation potential, and wavelength are still required before its empirical properties can be established. From a theoretical standpoint, its nature remains a mystery.

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I am grateful for the efficiency of the staff of the University Library, St. Andrews, in procuring the scientific literature which formed the basis of the discussion in the first two chapters; and I owe a special debt of thanks to Miss Joan Lawson for her conscientious and intelligent typing of this thesis.

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ON THE RED SHIFT OF THE SOLAR LINES

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Abstract. — *A rediscussion of the observations by* M. Adam *of* 14 *selected solar lines of mean wavelength* 6 100 Å.

1) *The fact that at the centre of the Sun the observed red shifts range from* 0.4×10^{-3} Å *to* 9.8×10^{-3} Å *and near the limb from* 7×10^{-3} Å *to* 19.2×10^{-3} Å — *the constant relativistic red shift should equal* 12.9×10^{-3} Å — *is due to systematic differences of wavelength from line to line. Eeach line must be treated individually. If the observed red shifts are represented as a superposition of Doppler effects (due to radial currents) upon the relativistic red shift the representation is very unsatisfactory.*

2) *The observations are however, perfectly represented by the formula :*

$$\Delta\lambda_r = X_r + Y . f(\theta, R/l_0)$$

The résulting values of Y are practically identical for all 14 *lines and equal to* 2.04×10^{-3} Å $\pm$ 0.04, *if the function* $f(\theta, R/l_0)$ *measuring the geometrical length of path which the light has travelled through the solar atmosphere is calibrated by using Evershed's observations at the Sun's limb.*

The values of X_r *range from* — 3×10^{-3} Å *to* $+ 6 \times 10^{-3}$ Å, *and are constant (within the range of accidental errors) across the disk.*

Резюме. — Новое изучение наблюдений 14 избранных солнечных линий средней длины волны 6100 Å, сделанных мисс М. Адам в 1946 г.

1) Тот факт, что в центре солнечного диска наблюденное красное смещение заключается в пределах от 0,4 $\times$ 10^{-3} Å до 9,8 $\times$ 10^{-3} Å, а вблизи края диска от 7 $\times$ 10^{-3} Å до 19,2 $\times$ 10^{-3}Å (откуда постоянное релятивистское красное смещение будет равным 12,9 $\times$ 10^{-3} Å), происходит вследствие систематической разности длин волн от линии к линии. Поэтому каждая линия должна быть рассматриваема отдельно. Представление наблюденных красных смещений, как наложения допплеровского смещения, происходящего от радиальных течений, на релятивистское красное смещение, является весьма неудовлетворительным.

2) Наблюдения, однако, могут быть превосходно представлены следующей формулой :

$$\Delta\lambda_r = X_r + Y . f\left(\theta, \frac{R}{l_0}\right)$$

Значения Y являются практически одинаковыми для всех 14 линий и равны 2,04 $\times$ 10^{-3} Å $\pm$ 0,04, если функция $f\left(\theta, \frac{R}{l_0}\right)$, дающая геометрическую длину пути, который свет проходит в солнечной атмосфере, оценена на основании наблюдений солнечного диска, сделанных Эвершедом. Остающиеся X_r заключаются между — 3 $\times$ 10^{-3} Å и + 6 $\times$ 10^{-3} Å и являются постоянными внутри всего диска (в пределах случайных ошибок).

Despite the fact that more than 40 years ago EINSTEIN derived from the principle of equivalence — many years before he was able to formulate the field equations of the general theory of relativity — that all solar lines should reveal a general red shift relative to the corresponding spectral lines produced by a terrestial light source, astronomers have not been able to decide whether this effect exists or not ; not because it is so small that observational errors make this decision impossible, but because the complexity of light emission on the surface of the Sun makes it extremely difficult to disentangle the various effects which influence the wavelength at which a solar line is produced. In addition, one other fact has, beyond doubt, delayed the decision pro or contra EINSTEIN's prediction ; namely, the tendency of some observers to approach the problem from the pre-established view that the effect *must* be existent. Consequently, their duty was only to explain why the solar lines do not reveal clearly the expected general red shift.

In the following investigation, based completely on already existing observational material, we have attempted to unravel the complex picture shown by the solar lines and to split up the observed displacements of the spectral lines into their various components without taking it for granted that all solar wavelengths must be influenced by the predicted gravitational red shift.

The observational data used are the $7 \times 14 = 98$ wavelength displacements of 14 selected solar lines in the restricted range of wavelength from 6 013.5 Å to 6 270.2 Å, measured by Miss M. ADAM in 1946 [1]. Every line was observed at 7 points along one radius of the sun. Since in a preceding publication by one of the authors [2*a*] it has already been shown that the change of wavelength is the same in whatever direction one proceeds from the centre to the limb of the Sun — naturally after the influence of the rotation of the Sun has been accounted for — it is sufficient to confine the observations to one radius, as Miss ADAM has done (1). It is, however, absolutely essential that the observations should comprise as many points on a radius as possible, for the increase of the observed shifts along every radius is a significant feature of the whole problem.

The vast amount of observations accumulated by ST. JOHN [3] has therefore become of lesser value, since this material is restricted almost completely to observations of the red shifts at the centre of the solar disk. The few observations made near to the Sun's limb are not sufficient for deriving the law according to which the observed red shifts increase towards the limb (instead of remaining constant as they should if the relativistic red shift alone were prevailing). Yet it is taken for granted that the observed increase of the red shift towards the limb follows a Cosine law. On this assumption is based the hypothesis, that, due to the superposition of Doppler effects of radial currents in the solar atmosphere,

(1) Miss ADAM actually measured the displacements at equal distances from the centre, along the polar diameter ; thus eliminating effects of solar rotation.

the constant relativistic red shift does not become manifest. It was shown, however, by FREUNDLICH and collaborators [2a] that the Cosine law does *not* represent the observed increase towards the limb.

For similar reasons the observational material provided by MEGGERS, BURNS, and their collaborators [4] has been disregarded. This is based on observations of integrated solar light, using a solar image of only 3.5 m/m diameter. These data would be of use in the present rediscussion only if the observed displacements had proved to be independent of the position on the Suns surface.

According to EINSTEIN'S theoretical prediction, all solar lines should be shifted towards the red end of the spectrum according to the formula : $\frac{\Delta\lambda}{\lambda} = \frac{\Delta\psi}{c^2}$ where $\Delta\psi$ denotes the difference of the gravitational potential between the Sun's and the Earth's surface ; the numerical value of the right-hand side is equal to 2.12×10^{-6}. Thus the lines observed by Miss ADAM, having the average wavelength $\bar{\lambda} = 6\,100$ Å should reveal a constant displacement of 12.9×10^{-3} Å. It will be shown later that the probable error of a single line position does not exceed $\pm\ 2 \times 10^{-3}$ Å ; hence the predicted shift is well within the limits of accuracy of observations.

We wish to emphasise that this rediscussion is by no means meant to be a cloaked criticism of Miss ADAM's work. We approach the same problem from a different direction, and we are deeply obliged to Miss ADAM for providing us with such valuable observational material which quite definitely is, to date, the most accurate set of measurements available from which to decide whether solar observations reveal the predicted relativistic red shift or not. Our results corroborate the main conclusion of Miss ADAM, but we drop the restrictive remark that only " over 80-90 per cent of the solar disk " an unknown effect must neutralize the relativistic shift. Our dissection of the material reveals new facts hitherto not known, concerning the origin of the red shifts actually observed. Table I, reproduced from Miss ADAM's paper (1948), summarises the results on which our investigation is based. The centre shifts are the mean of 11 individual observations ; the others are the mean of 6 individual observations. θ is the angle made by the outward direction of a solar radius with the line of sight to the observer.

A close inspection of Table I reveals the fact that the fluctuations of the observed mean displacements at the centre of the disk cannot be due to accidental errors.

This result is suggested by Table II, in which the corresponding line shifts of 5 pairs of lines, included in ADAM's data, are compared ; the relative differences are seen to be constant for all 7 disk positions to within $\pm\ 2 \times 10^{-3}$ Å.

The same result holds for all line pairs which can be selected from ADAM's data This implies that the observational uncertainty in the mean differential displacements of any one line lies well below 2×10^{-3} Å. Hence the much larger

TABLE I

MEAN OBSERVED SHIFTS FOR INDIVIDUAL LINES (Unit of $\Delta\lambda_i = 1 \times 10^{-3}$ Å)

DISK POSITION			0	1	2	3	4	5	6
SIN θ			0.000	0.577	0.769	0.884	0.918	0.959	0.984
COS θ			1.000 { 1.000 1.000	.817 { .805 .828	.639 { .619 .658	.468 { .436 .498	.398 { .357 .434	.282 { .219 .333	.181 { .000 .255
WAVELENGTH	El.	Int.	$\Delta\lambda_0$	$\Delta\lambda_1$	$\Delta\lambda_2$	$\Delta\lambda_3$	$\Delta\lambda_4$	$\Delta\lambda_5$	$\Delta\lambda_6$
—	—	—	—	—	—	—	—	—	—
6 013.5	Mn	6	+ 2.8	3.8	3.3	4.2	4.3	5.0	11.2
6 016.6	Mn	6	5.1	5.8	3.8	5.0	6.3	9.8	12.3
6 021.8	Mn	6	6.5	7.0	6.0	7.5	7.0	10.0	14.2
6 024.1	Fe	7	2.5	1.5	4.0	2.8	4.5	7.7	9.0
6 027.1	Fe	4	9.6	8.7	8.8	7.0	11.8	14.7	16.8
6 042.1	Fe	3	7.6	9.3	9.5	11.3	11.0	13.2	18.5
6 056.0	Fe	5	2.5	3.5	3.0	3.5	6.0	7.3	11.5
6 065.5	Fe	7	7.2	7.8	7.8	9.7	10.2	12.0	14.5
6 108.1	Ni	6	1.3	— 0.3	— 0.7	1.3	2.7	3.0	8.3
6 122.2	Ca	10	6.3	7.5	7.3	9.5	10.7	11.2	14.0
6 200.3	Fe	6	5.5	5.7	7.3	7.2	6.8	10.5	13.8
6 219.3	Fe	6	3.2	3.8	4.7	2.8	7.0	7.8	10.8
6 265.1	Fe	5	9.8	10.3	9.0	10.0	12.2	12.2	19.2
6 270.2	Fe	3	0.4	0.3	— 0.7	0.5	0.3	3.2	7.0

TABLE II

MEAN DIFFERENCES BETWEEN LINES (Unit : 1×10^{-3} Å)

DISK POSITION			0	1	2	4	4	5	6	MEAN	PROBABLE ERROR
SIN θ			0.000	.577	.769	.884	.918	.959	.984	DIFFERENCE	OF SINGLE DIFFERENCE
λ	El.	Int.									
—	—	—	—	—	—	—	—	—	—	—	—
6 013.5	Mn	6	+ 2.8	3.8	3.3	4.2	4.3	5.0	11.2		
6 016.6	Mn	6	+ 5.1	5.8	3.8	5.0	6.3	9.8	12.3		
		Diff.	2.3	2.0	0.5	0.8	2.0	4.8	1.1	*1.9*	± 1.0
6 013.5	Mn	6	+ 2.8	3.8	3.3	4.2	4.3	5.0	11.2		
6 021.8	Mn	6	+ 6.5	7.0	6.0	7.5	7.0	10.0	14.2		
		Diff.	3.7	3.2	2.7	3.3	2.7	5.0	3.0	*3.4*	± 0.5
6 108.1	Ni	6	+ 1.3	— 0.3	— 0.7	1.3	2.7	3.0	8.3		
6 122.2	Ca	10	+ 6.3	7.5	7.3	9.5	10.7	11.2	14.0		
		Diff.	5.0	7.8	8.0	8.2	8.0	8.2	5.7	*7.3*	± 0.9
6 122.2	Ca	10	+ 6.3	7.5	7.3	9.5	10.7	11.2	14.0		
6 270.2	Fe	3	+ 0.4	0.3	— 0.7	00.5	0.3	3.2	7.0		
		Diff.	5.9	7.2	8.0	9.0	10.4	8.0	7.0	*7.9*	± 1.0
6 265.1	Fe	5	+ 9.8	10.3	9.0	10.0	12.2	12.2	19.2		
6 270.2	Fe	3	+ 0.4	0.3	— 0.7	0.5	0.3	3.2	7.0		
		Diff.	9.4	10.0	9.7	9.5	11.9	9.0	12.2	*10.2*	± 0.9

differences between the centre displacements of the lines cannot be regarded as of accidental nature ; the shifts should not be averaged out indiscriminately at each disk position.

A statistical study of the complete material which Miss ADAM kindly put at our disposal, has fully justified the above conclusion. It has been found from an analysis of 658 independent observations that the probable error of a single observation is $\pm 1.8 \times 10^{-3} < \pm 2 \times 10^{-3}$ Å. Since the central values in Table I are the mean values of 11, the others of 6 independent observations, the probable error of a value in Table I is equal to

$$\pm \frac{1.8}{\sqrt{11}} \times 10^{-3} \text{ Å} = \pm 0.5 \times 10^{-3} \text{ Å},$$

for observations at the centre of the Sun ; and

$$\pm \frac{1.8}{\sqrt{6}} \times 10^{-3} \text{ Å} = \pm 0.7 \times 10^{-3} \text{ Å}$$

for measurements of the displacements across the disk. Hence fluctuations of not more than about $\pm 1 \times 10^{-3}$ Å in the mean displacements due to accidental errors alone are to be expected. Thus the much larger differences of the observed red shifts for the various lines at the centre of the Sun's disk are real ; that is, they are due either to physical effects or to systematic errors in wavelength. Every spectral line, for each of which there are 7 independent values of displacements, for θ_i ($i = 0, 1, \ldots 6$), must therefore be treated separately.

The following questions require to be answered :

(*a*) What is the nature of the increase of the displacements towards the Sun's limb which is shown by all lines without exception ?

(*b*) In particular, is it possible, without distorting the observations, to represent them by a superposition of a constant general red shift — equal to 12.9×10^{-3} Å for the material under consideration — plus a limb effect which usually has been assumed to have the character of a Doppler effect produced by radial currents in the solar atmosphere ?

(*c*) What is the nature of the line displacements in the centre of the Sun ranging from practically zero to 3/4 times the predicted relativistic shift ?

Fig. 1 shows for the observational material under consideration the observed course of the displacements from the centre to the limb for the smallest central red shift $= + 0.4 \times 10^{-3}$ Å, for the largest value $+ 9.8 \times 10^{-3}$ Å, and finally for two arbitrarily chosen intermediate lines. The shaded area, on the other hand, indicates the course which the observed displacements should follow if the deviations from a constant general red shift were produced by the superposition of Doppler effects decreasing towards the limb according to a Cosine law. In this latter case the relativistic red shift should appear as the *limiting* value towards

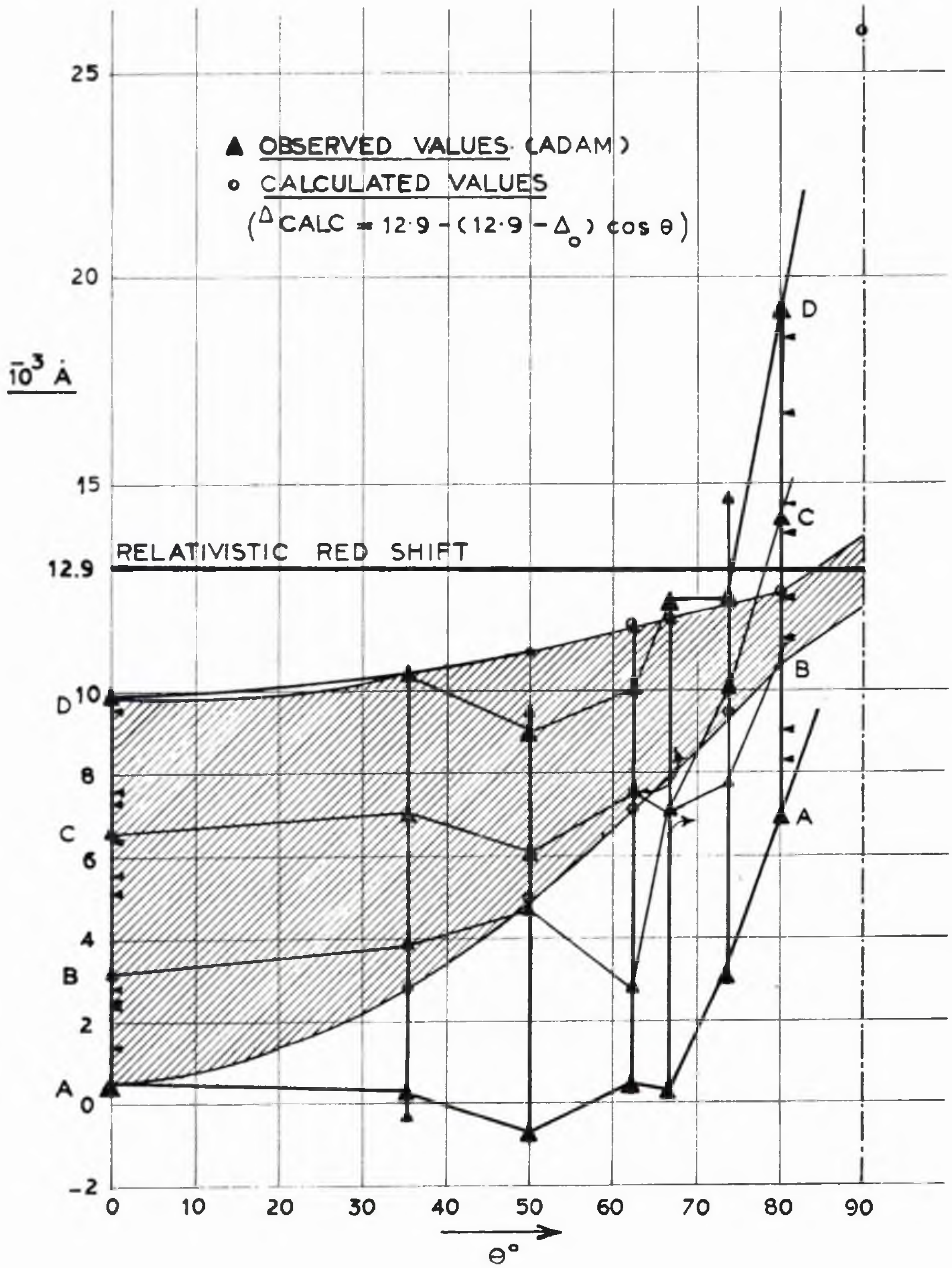

FIG. 1.

which all observed displacements *converge* in the limit $\theta = 90^o$. The graph (fig. 1), however, reveals :

i) The increase of the displacements towards the limb does not follow the Cosine law. This same result had been obtained years before from the investigations at Potsdam [2*a*]. Both sets of observations are well represented by a secant formula, even though different wavelength regions are employed.

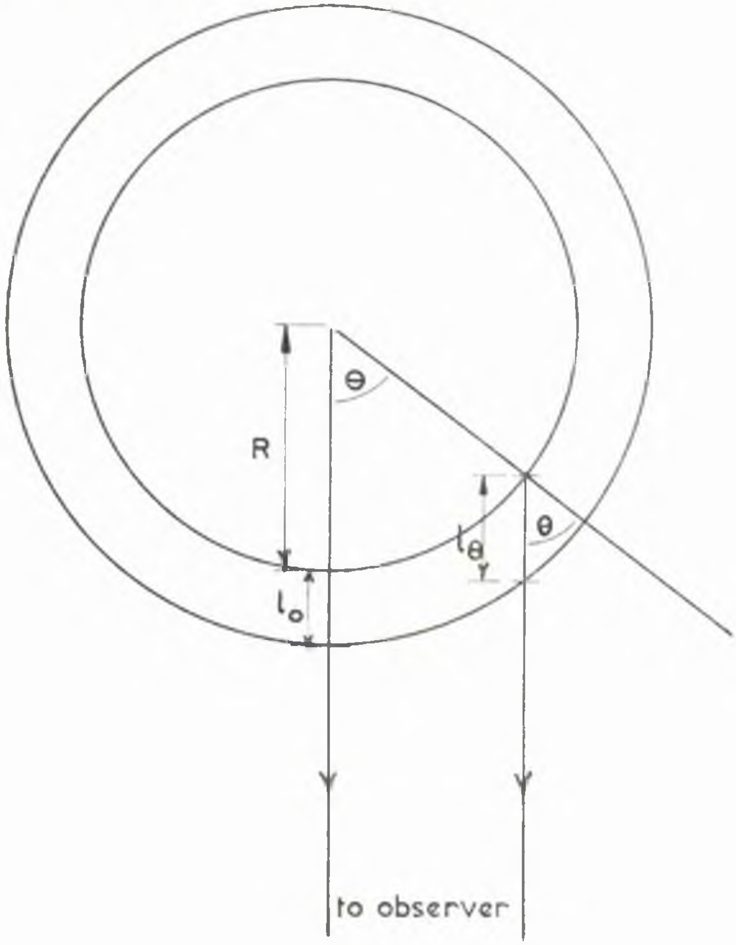

FIG. 2.
(Diagram illustrating notation)

ii) The smallest central displacement, 0.4×10^{-3} Å, leads to the smallest value for $\theta = 80°$; similarly, the largest central value, 9.8×10^{-3} Å yields the largest limb value. In fact, the seven of the fourteen values which exceed the arithmetic mean value at the centre ($\theta = 0°$) correspond to the seven values which exceed the arithmetic mean near the limb (at $\theta = 80°$). The mean differences $(\Delta\lambda_i - \Delta\lambda_0)$ for any line, where $i = 1, 2, \ldots 6$, are constant to within the limit of accidental errors.

iii) The observed displacements do not converge towards a limiting value near the limb. The range of values for $\theta = 80°$ is even larger than the range in the centre.

These three facts leave no doubt that a " limb effect " is superimposed upon the central displacements ; however, one which is not — as is generally supposed — confined to a region of the disk close to the limb, but which operates throughout the full range of values of θ from $\theta = 0^o$ to $\theta = 90^o$.

The next problem to consider is : Since the increase of the displacements towards the limb does not follow the Cosine law, which law does govern this increase ?

In a preceding paper (1954) [2*b*] it has been shown that a secant formula represents the observations very well ; a secant formula is, similarly, a good approximation to the ratio l_θ/l_0 i.e. $l_\theta = l_0 \sec\theta$ where l_0 denotes the depth from which the line originates at $\theta = 0$. The correct formula for l_θ/l_0 must take into account the spherical shape of the Sun, and the finite path length at $\theta = 90^o$: it is a function not only of θ but of R/l_0 too, where R is the radius of the Sun. It has the form : (See fig. 2)

$$f(\theta, R/l_0) = -\frac{R}{l_0}\cos\theta + \sqrt{\left(2\frac{R}{l_0}+1\right) + \left(\frac{R}{l_0}\cos\theta\right)^2}.$$

TABLE III

COMPARISON OF $f(\theta, R/l_0)$ WITH SEC θ

DISK POSITION	0	1	2	3	4	5	6
—	—	—	—	—	—	—	—
Cos θ	1.000	0.817	0.639	0.468	0.398	0.282	0.181
Sec θ	1.000	1.224	1.565	2.137	2.513	3.546	5.522
$f(\theta, 10^4)$	1.000	1.224	1.565	2.136	2.512	3.544	5.516
$f(\theta, 10^2)$	1.000	1.221	1.554	2.100	2.449	3.362	4.887
$f(\theta, 50)$	1.000	1.218	1.543	2.066	2.394	3.215	4.474

The third decimal place is not significant on account of the uncertainty in the value of cos θ. The secant approximation is seen to apply very well over the range of θ covered by the observations ; i.e. $0 \leqslant \theta \leqslant 80^o$; in the range $80^o \leqslant \theta \leqslant 90^o$ however, f and sec θ diverge rapidly until at $\theta = 90^o$, $f = \sqrt{2\frac{R}{l_0}+1}$ while sec $\theta \to \infty$.

The observed red shift $\Delta\lambda_r$ of any line r is represented with very great accuracy by the law :

$$\Delta\lambda_r = X_r + Y_r . f(\theta_i, R/l_0) \qquad \begin{matrix} i = 0, 1, 2 \ldots . 6 \\ r = 1, 2, 3 \ldots . 14 \end{matrix}$$

for a suitable choice of R/l_0. We chose, provisionally, $R/l_0 = 10^4$ or $l_0 \simeq 70$ km, which is equivalent to assuming that the red shift is produced within the solar reversing layer. Least squares solutions have been carried out for all the lines comprising ADAM's data : the most probable values of X_r and Y_r *in units of* 10^{-3} Å are given in Table IV.

TABLE IV

SOLUTIONS OF $\Delta\lambda_r = X_r + Y_r f(\theta, 10^4)$ (Unit : 1×10^{-3} Å)

WAVELENGTH	El.	Int.	X	Y
6 013.5	Mn	6	+ 0.82	+ 1.66
6 016.6	Mn	6	2.61	1.74
6 021.8	Mn	6	3.92	1.77
6 024.1	Fe	7	0.57	1.62
6 027.1	Fe	4	6.50	1.88
6 042.1	Fe	3	5.77	2.26
6 056.0	Fe	5	0.39	1.98
6 065.5	Fe	7	5.71	1.66
6 108.1	Ni	6	— 1.75	1.66
6 122.2	Ca	10	5.18	1.70
6 200.3	Fe	6	3.57	1.83
6 219.3	Fe	6	1.48	1.70
6 265.1	Fe	5	6.98	1.98
6 270.2	Fe	3	— 2.10	1.51

$Y = + 1.78 \pm 0.03$

The X_r are absolute quantities, the values of which will include the effect of unknown errors involved in comparing the solar wavelengths with the terrestial standards and physical effects influencing the wavelength of a line. Their possible origin will be discussed later.

The most significant feature of the above analysis is the remarkable constancy of the Y_r. So closely do they congregate about the mean value $Y = 1.78 \times 10^{-3}$ Å, — the probable error is less than 2 % of the mean value — that Y may replace the Y_r in analysing any of the 14 lines. The value of Y extrapolated formerly from the B-and O-stars had been 1.7×10^{-3} Å [2*b*]. The question which arises naturally from this discussion is : Are other observations available which will help fix the value of R/l_0 ? Such a possibility is given if we make use of EVERSHED's results [5], supported by observations made at Mount Wilson that the (limb-arc) displacements of a group of lines of mean wavelength 6 200 Å measured at the limb of the Sun ($\theta \simeq 90°$) is ~ 0.025 Å.

For

$$R/l_0 = 10^4, \quad f(90°, R/l_0) = \sqrt{2\frac{R}{l_0} + 1} = 141.4.$$

Hence $Y . f\,(90°, 10^4) = 0\ 252$ Å ; this is exactly 10 times the value observed by EVERSHED. A better representation is therefore needed ; $R/l_0 = 10^2$ gives a very close agreement with EVERSHED's observations. The results of the least-squares solutions are given in Table V. As before, the unit for X_r and Y_r is 10^{-3} Å.

TABLE V

SOLUTIONS OF $\Delta\lambda_r = X_r + Y_r f(\theta, 100)$ (Unit : 1×10^{-3} Å)

WAVELENGTH	El.	Int.	X_r	Y_r
6 013.5	Mn	6	0.51	1.89
6 016.6	Mn	6	2.23	2.00
6 021.8	Mn	6	3.53	2.04
6 024.1	Fe	7	0.21	1.87
6 027.1	Fe	4	6.13	2.15
6 042.1	Fe	3	5.31	2.59
6 056.0	Fe	5	— 0.02	2.27
6 065.5	Fe	7	5.34	1.92
6 108.1	Ni	6	— 2.05	1.88
6 122.2	Ca	10	4.78	1.97
6 200.3	Fe	6	3.20	2.09
6 219.3	Fe	6	1.13	1.95
6 265.1	Fe	5	6.65	2.23
6 270.2	Fe	3	— 2.36	1.71

$Y = + 2.04 \pm 0.04$

The constancy of the Y values is seen to be preserved.

The accuracy of the overall representation, as measured by the sum of squares of residuals, deteriorates only very slightly, the sum increasing from 86 to 95. In this case $Y. f(90^{\circ}, 10^2) = 2.04 \times 10^{-3} \times 14.14 = + 0\ 029$ Å, which is somewhat larger than the observed value. In view, however, of the uncertainty of the actual amount which the red shift attains at the limb, this difference is not significant. For the same reason the unknown contribution of a mean X_r which for the group of lines at $\bar{\lambda} = 6\ 100$ Å amounts to only a few thousands of an Angström, (See Table V) has been disregarded.

A third set of solutions has also been carried out for $R/l_0 = 50$ for which $Y. f(90^{\circ}, 50) = 0\ 0225$ Å ; this is still of the correct order of magnitude but already slightly smaller than the observed value. Again the overall representation is less good (1).

(1) A small increase in the sum of squares of residuals, however, may be due to the inaccuracy of the measures, the uncertainty in Cos θ and to a possible physical effect of unknown origin but only of secondary importance, which shows itself in a tendency for the observed displacements to diminish slightly before increasing rapidly near the limb. (Inspect Table I for this effect.) It need not necessarily mean that the choice of R/l_0 is less correct.

The most sensitive test for the optimum value of R/l_0 is the compatibility of the $Y . f(\theta, R/l_0)$ term with EVERSHED's observed value at the limb. The accuracy of this test is limited by the observational uncertainties. Another test is offered by comparing the calculated values of $(\Delta\lambda_6 - \Delta\lambda_0)$ for different choices of R/l_0, with the mean observed value of $(7.9 \pm 0.2)\ 10^{-3}$ Å resulting from ADAM's data.

TABLE VI

DEPENDENCY OF LIMB INCREASE ON R/l_0 (Unit : 1×10^{-3} Å)

R/l_0	$f(80)$	$f(80) - f(0)$	$Y[f(80) - f(0)] = (\Delta\lambda_6 - \Delta\lambda_0)$	PROBABLE ERROR OF $(\Delta\lambda_6 - \Delta\lambda_0)$
—	—	—	—	—
10,000	5.516	4.516	$1.78 \times 4.516 = +\ \mathit{8.0}$	$\pm\ 0.03 \times 4.516 = \pm\ \mathit{0.1}$
100	4.930	3.930	$2.04 \times 3.930 = +\ \mathit{8.0}$	$\pm\ 0.04 \times 3.930 = \pm\ \mathit{0.2}$
50	4.474	3.474	$2.25 \times 3.474 = +\ \mathit{7.8}$	$\pm\ 0.04 \times 3.474 = \pm\ \mathit{0.1}$

This test is obviously not sensitive ; it is, however, a significant fact that the observations give exactly the value 7.9, characteristic of the function $f(\theta, R/l_0)$ over a wide range of values for R/l_0.

From the foregoing discussion it follows that $R/l_0 = 10^2$ is of the correct order. This result is of great importance, as it implies that the red shifts, increasing to the limb due to the interaction term, are produced within the chromosphere, i.e. within the intense radiation field close to the Sun. The centre-limb increase is fully accounted for by this interaction term :$Y . f(\theta, R/l_0)$.

Finally let us consider whether *other* possibilities of representing the observational data exist.

In former investigations it has been assumed that the predicted red shift $\Delta\lambda/\lambda = 2.12 . 10^{-6}$, constant over the whole disk of the Sun, must be inherent in all observations of solar lines, and since such a constant displacement of all lines was not observed, all efforts were concentrated upon proving that radial (up-and-down) currents of sufficient strength exist in the solar atmosphere, capable of producing the complex picture actually observed, viz : a red shift too small in the centre of the Sun, attaining the theoretically predicted value near the limb. All discussions were focussed upon the question whether the Doppler effects, introduced in order to account for the reduction of the observed red shifts, were compatible with the existing model of the structure of the solar atmosphere, (*cf.* [1] and [6]). However, it was known from the beginning of these investigations that near to the Sun's limb the observed displacements are considerably larger than the expected limiting value, so that in addition, the *ad hoc* hypothesis of a " limb effect " (1) had to be made to account for this discrepancy. We have no

(1) In the sense used by EVERSHED, meaning an excess over the relativistic value.

wish to enter into a detailed critical discussion of the possible existence of regular radial currents in the solar atmosphere ; for to find a conclusive answer to this question will not be easy. The obvious turbulent appearance of the Sun's surface (granulation), however, does not support this view.

Despite the indisputable facts that the increase of the line displacements towards the limb does not follow the Cosine law, and that the observed red shifts do not converge near the limb towards a defined limiting value, the observational material of Miss ADAM has also been tested by us from the point of view whether it permits, with satisfactory accuracy, another representation according to the former hypothesis — i.e. that of a superposition of Doppler effects upon a constant red shift. Two sets of least-square solutions have been made ; in the first, the seven values of the red shifts observed for every line for θ_i $(i = 0, 1, 2...6)$ are represented by the formula :

$$\Delta\lambda_r = A_r - B_r \cos\theta \qquad r = 1, 2, 3 \ldots . 14.$$

If an identical value of " A_r " had resulted for all lines, this would have been a criterion in favour of such a representation. Secondly, the observations for every line were represented by the formula :

$$\Delta\lambda_r = 12.9 \times 10^{-3} - B_r \cos\theta$$

based on the fact that the general theory of relativity prescribes strictly the constant value : $A_r = 12.9 \times 10^{-3}$ Å for the red shift of all the spectral lines used in our investigation. The outcome of these least-square solutions are reproduced in the tables VII and VIII.

In the first case, (Table VII) there is no indication whatsoever of a constant identical " A " for all 14 lines. The values of " A_r " fluctuate *even more* than the originally observed displacements at the centre of the Sun.

In the second case, the resulting representation of the observations (table VIII) is definitely unsatisfactory and very much inferior to the representation by the formula :

$$\Delta\lambda_r = X_r + 2.04 \times 10^{-3}.f(\theta, 10^2) \quad r = 1, 2, 3 \ldots . 14.$$

In Table VIII the sum of the squares of the residuals rises to 484 (1), while the corresponding value obtained from the residuals in Table X, is equal to 95 ; 48 among the 98 residuals exceed 1.5×10^{-3} Å, 18 exceeding even 3.5×10^{-3} Å ; while only 9 residuals in Table X exceed 1.5×10^{-3} Å, and the average absolute value is below 1×10^{-3} Å — viz : 0.7×10^{-3} Å.

(1) By using the B_r values resulting for each line when calculating the residuals of

$$\Delta\lambda_r = 12{,}9.10^{-3} - B_r \cos\theta,$$

we are keeping the sum of squares to a minimum for this representation. If, as should have been done, the residuals are calculated with a mean B_r the accuracy of the representation becomes very much worse,

TABLE VII

ALTERNATIVE SOLUTIONS OF $\Delta\lambda_r$ FROM § 5 (Unit : 1×10^{-3} Å)

WAVELENGTH	$\Delta\lambda_r = A_r - B_r \cos\theta$ A_r	B_r	$\Delta\lambda_r = 12.9 - B_r \cos\theta$ B_r
—	—	—	—
6 013.5	+ 8.49	+ 6.40	+ 12.39
6 016.6	10.62	6.68	9.78
6 021.8	11.95	6.49	7.78
6 024.1	8.60	7.20	13.04
6 027.1	15.19	7.25	4.14
6 042.1	17.04	10.11	4.49
6 056.0	10.04	8.48	12.38
6 065.5	14.17	7.75	6.01
6 108.1	5.76	6.09	15.85
6 122.2	14.03	8.26	6.72
6 200.3	12.47	7.85	8.43
6 219.3	9.87	7.47	11.58
6 265.1	16.03	7.48	3.23
6 270.2	4.61	5.37	16.63

TABLE VIII

TABLE OF RESIDUALS ($v_i = 12.9 - B_r \cos\theta$) (Unit : 1×10^{-3} Å)

DISK POSITION	0	1	2	3	4	5	6
Cos θ	1.000	.817	.639	.468	.398	.282	.181
WAVELENGTH	v_0	v_1	v_2	v_3	v_4	v_5	v_6
—	—	—	—	—	—	—	—
6 013.5	+ 2.3	+ 1.0	— 1.7	— 2.9	— 3.7	— 4.4	+ 0.5
6 016.6	+ 2.0	+ 0.9	— 2.9	— 3.3	— 2.7	— 0.3	+ 1.2
6 021.8	+ 1.4	+ 0.5	— 1.9	— 1.8	— 2.8	— 0.7	+ 2.7
6 024.1	+ 2.6	— 0.7	— 0.6	— 4.0	— 3.2	— 1.5	— 1.5
6 027.1	+ 0.8	— 0.8	— 1.5	— 4.0	+ 0.5	+ 3.0	+ 4.7
6 042.1	— 0.8	+ 0.1	— 0.5	+ 0.5	— 0.1	+ 1.6	+ 6.4
6 056.0	+ 2.0	+ 0.7	— 2.0	— 3.6	— 2.0	— 2.1	+ 0.8
6 065.5	+ 0.3	— 0.2	— 1.3	— 0.4	— 0.3	+ 0.8	+ 2.7
6 108.1	+ 4.2	— 0.3	— 3.5	— 4.2	— 3.9	— 5.4	— 1.7
6 122.2	+ 0.1	+ 0.1	— 1.3	— 0.3	+ 0.5	+ 0.2	+ 2.3
6 200.3	+ 1.0	— 0.3	— 0.2	— 1.8	— 2.7	0.0	+ 2.4
6 219.3	+ 1.9	+ 0.4	— 0.8	— 4.7	— 1.3	— 1.8	0.0
6 265.1	+ 0.1	0.0	— 1.8	— 1.4	+ 0.6	+ 0.2	+ 6.9
6 270.2	+ 4.1	+ 1.0	— 3.0	— 4.6	— 6.0	— 5.0	— 2.9

Sum of mean square residuals = 484

TABLE IX

X_r VALUES

$X_r = \Delta\lambda_{obs} - 2.04\, f(\theta, 10^2)$ (Unit : 1×10^{-3} Å)

DISK POSITION	0	1	2	3	4	5	6	MEAN
SIN θ	0.000	0.577	0.769	0.884	0.918	0.959	0.984	
	2.0	2.5	3.2	4.3	5.0	6.9	10.0	X_r
—	—	—	—	—	—	—	—	—
6 013.5 Å	0.8	1.3	0.1	— 0.1	— 0.7	— 1.9	1.2	+ 0.1
6 016.6	3.1	3.3	0.6	0.7	1.3	2.9	2.3	+ 2.0
6 021.8	4.5	4.5	2.8	3.2	2.0	3.1	4.2	+ 3.5
6 024.1	0.5	— 1.0	0.8	— 1.5	— 0.5	0.8	— 1.0	— 0.3
6 027.1	7.6	6.2	5.6	2.7	6.8	7.8	6.8	+ 6.2
6 042.1	5.6	6.8	6.3	7.0	6.0	6.3	8.5	+ 6.6
6 056.0	0.5	1.0	— 0.2	— 0.8	1.0	0.4	1.5	+ 0.5
6 065.5	5.2	5.3	4.6	5.4	5.2	5.1	4.5	+ 5.0
6 108.1	— 0.7	— 2.8	— 3.9	— 3.0	— 2.3	— 3.9	— 1.7	— 2.6
6 122.2	4.3	5.0	4.1	5.2	5.7	4.3	4.0	+ 4.7
6 200.3	3.5	3.2	4.1	2.9	1.8	3.6	3.8	+ 3.3
6 219.3	1.2	1.3	1.5	— 1.5	2.0	0.9	0.8	+ 0.9
6 265.1	7.8	7.8	5.8	5.7	7.2	5.3	9.2	+ 7.0
6 270.2	— 1.6	— 2.2	— 3.9	— 3.8	— 4.7	— 3.7	— 3.0	— 3.3

TABLE X

TABLE OF RESIDUALS

$x_i = \Delta\lambda_i - [2.04\, f(\theta, 10^2) + X_r]$ (Unit : 1×10^{-3} Å)

DISK POSITION	0	1	2	3	4	5	6
SIN θ	0.000	.577	.769	.884	.918	.959	.984
WAVELENGTH	x_0	x_1	x_2	x_3	x_4	x_5	x_6
—	—	—	—	—	—	—	—
6 013.5 Å	+ 0.3	+ 0.8	— 0.4	— 0.6	— 1.2	— 2.4	+ 0.7
6 016.6	+ 0.9	+ 1.1	— 1.6	— 1.5	— 0.9	+ 0.7	+ 0.1
6 021.8	+ 1.0	+ 1.0	— 0.7	— 0.3	— 1.5	— 0.4	+ 0.7
6 024.1	+ 0.3	— 1.2	+ 0.6	— 1.7	— 0.7	+ 0.6	— 1.2
6 027.1	+ 1.5	+ 0.1	— 0.5	— 3.4	+ 0.7	+ 1.7	+ 0.7
6 042.1	+ 0.3	+ 1.5	+ 1.0	+ 1.7	+ 0.7	+ 1.0	+ 3.2
6 056.0	+ 0.5	+ 1.0	— 0.2	— 0.8	+ 1.0	+ 0.4	+ 1.5
6 065.5	— 0.1	0.0	— 0.7	+ 0.1	— 0.1	— 0.2	— 0.8
6 108.1	+ 1.4	— 0.7	— 1.8	— 0.9	+ 0.2	— 1.8	+ 0.4
6 122.2	— 0.5	+ 0.2	— 0.7	+ 0.4	+ 0.9	— 0.5	— 0.8
6 200.3	+ 0.3	0.0	+ 0.9	— 0.3	— 1.4	+ 0.6	+ 0.1
6 219.3	+ 0.1	+ 0.2	+ 0.4	— 2.6	+ 0.9	— 0.2	— 0.3
6 265.1	+ 1.1	+ 1.1	— 0.9	— 1.0	+ 0.5	— 1.4	+ 2.5
6 270.2	+ 0.8	+ 0.2	— 1.5	— 1.4	— 2.3	— 1.3	— 0.6

Sum of squares of residuals = 95

Therefore, the answer to question (*b*) §3, is that it is impossible to represent satisfactorily the observations by the superposition of Doppler effects upon the relativistic red shift.

The third, and last, question to be considered concerns the nature of the centre shifts. As a result of the preceding analysis we now know that the increase towards the limb of *all* 14 line displacements is the same and can be fully attributed to the effect of the interaction term $Y f(\theta)$. When this is subtracted from the observed red shifts, the X_r remain, the values of which vary across the disk only within the range of accidental errors.

Table IX illustrates the constancy of the 98 values $X_r = \Delta\lambda_{obs} - 2.04 f(\theta, 10^2)$; the deviations from the mean are as follows :

$$
\begin{array}{rl}
 & 69 < 0.001 \text{ Å} \\
 & 29 > 0.001 \text{ Å} \\
 & 3 > 0.002 \text{ Å} \\
\text{and only} & 1 > 0.003 \text{ Å.}
\end{array}
$$

These values are wholly attributable to accidental errors.

The X_r averaged over the θi ($i = 0, 1, 2 \ldots 6$) vary from -3×10^{-3} Å to $+7 \times 10^{-3}$ Å ; each tabulated value having, independent of its absolute amount, a probable error of about $\pm 0.2 \times 10^{-3}$ Å. The $\overline{X}_r$ will include systematic errors introduced through the comparison of both solar and terrestial wavelengths with the primary standard and also all effects due to pressure, atomic collision, etc. : they lie apparently within the possible limits of accuracy set by the comparison with the cadmium standard, for differences of as much as 0.010 Å have been found between these earlier and other more recent wavelength determinations.

Conclusions

The observed displacements of solar lines do not reveal the existence of a constant red shift of the amount predicted by the general theory of relativity. The increase of the displacements from the centre towards the limb does not corroborate the view that Doppler effects, produced by radial currents in the atmosphere, are superimposed upon a constant general red shift. Neither is the Cosine law satisfied, nor is any convergence towards a limiting value indicated.

If a representation by the formula :

$$\Delta\lambda_r = 12.9 \times 10^{-3} - B_r \cos\theta$$

is forced upon the observations, this representation becomes unsatisfactory, the sum of the squares of the residuals rising to 484, while this sum is reduced to 95 when the representation

$$\Delta\lambda_r = X_r + Y . f\left(\theta, \frac{R}{l_0}\right)$$

is chosen. No satisfactory physical interpretation of this last representation has yet been forthcoming. The presence of the X_r implies that the frequency with which a line emerges at a certain depth of the solar atmosphere, is slightly out of tune — probably due to systematic errors of the applied wavelengths, pressure effects and similar influences. Superimposed upon the X_r appears the interaction term : $Y \times f(\theta, R/l_0)$ depending on the geometrical path length traversed by the light in passing through the radiation field close to the Sun. Formerly [2*b*] the value of Y for the Sun had been predicted as equal to 1.7×10^{-3} Å from the red shifts of B and O-stars. The present more detailed study of Miss ADAM's data yields from 14 independent determinations with great accuracy $Y = 1.78 \times 10^{-3}$ Å if as before R/l_0 is chosen equal to 10^4. The discussion in §5 reveals, however, that a more probable value of R/l_0 is $\sim 10^2$, which corresponds to a value of $Y = 2.04 \times 10^{-3}$ Å $\pm\ 0.04.10^{-3}$. The existence of this interaction term seems to be beyond question.

The purpose of this paper has been to give an unbiased discussion of the observed data. We have therefore avoided all theoretical speculations which would only detract from the fundamental conclusions derived from our analysis. The theories of LINDHOLM and SPITZER, which are belived to account respectively for small red and violet displacements of the order of the X_r values, will be discussed later.

The relation between the shift of a line and its intensity — or better, line strength — which is a well-established feature of all measurements of solar wavelengths, has also not been included in this discussion. Any investigation of this correlation will be fruitless until more accurate observations are available.

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ON THE RED SHIFT OF THE SOLAR LINES, II

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Abstract. — *According to the first paper in these Annales on the red shift of the solar lines the interaction term, now denoted by $\Delta\lambda_i$, can be derived without any reference to terrestial wavelengths. Limb and centre-observations alone are needed. An extensive material of observations from the Mt. Wilson and other observatories is discussed in this second paper from the new point of view.*

The discussion, covering a few hundred lines belonging to various elements, yields two main results ; 1) the ratio $\frac{\Delta\lambda_i}{\lambda}$ is practically constant over a wide wavelength range from about 3 570 Å to 6 750 Å and 2) the observations reveal clearly a correlation between the value of $\Delta\lambda_i$ and the depth, i. e. temperature-level from which the line emerges.

Резюме. — Согласно первой статье, помещенной в этом журнале и трактующей вопрос о красном смещении солнечных линий, член взаимодействия, обозначаемый теперь через $\Delta\lambda_i$, может быть выведен без всяких ссылок на земные длины волн. Необходимы только наблюдения края и центра. Обширный наблюдательный материал Маунт-Вилсона и других обсерваторий обсуждается в этой второй статье с новой точки зрения.

Дискуссия охватывает несколько сот линий, принадлежащих различным элементам, и дает два основных вывода : 1) отношение $\frac{\Delta\lambda_i}{\lambda}$ является практически постоянным на широком участке длин волн (от 3 570 Å до 6 750 Å), и 2) наблюдения ясно обнаруживают зависимость между значением $\Delta\lambda_i$ и глубиной, т. е. температурным уровнем, на котором возникает данная линия.

In a preceding paper [1] we have shown that the change of the red shift of the solar lines from the centre to the limb in a spectral region around $\bar{\lambda} = 6\,100$ Å can be completely represented by the formula :

$$\Delta\lambda_{(r)} = X_{(r)} + Y . f(\theta, R/l_0)$$

for $0 \leqslant \theta \leqslant 90^o$. Here $X_{(r)}$ signifies a small shift characteristic of every individual line " r " and constant over the whole solar disk : this term shall from now on be denoted by $\Delta\lambda_c^{(r)}$. Also, $\Delta\lambda_i$ instead of Y shall now denote the *interaction displacement*, which has been found to be practically identical for every line within the restricted range of wavelengths previously employed. $f(\theta, R/l_0) = l_\theta/l_0$ is the relative increase in the length of path, traversed by the photon through the Sun's atmosphere. Hence :

$$(1) \qquad \Delta\lambda^{(r)}_{obs} = \Delta\lambda^{(r)}_{e} + \Delta\lambda_i \cdot f(\theta, R/l_0).$$

The existence of the interaction displacement $\Delta\lambda_i$ is, to date, the most significant result of our investigation. It appears therefore to be of particular interest to examine the universal character of $\Delta\lambda_i$ by establishing its variation with the wavelength λ. At present no other set of observations exists which, like the Oxford set, measures the change of the red shift along a solar radius up to a defined distance from the limb. However, observations of solar lines confined only to the limb and the centre of the Sun's disk suffice to determine the interaction displacement $\Delta\lambda_i$, since :

$$(2) \qquad \Delta\lambda_i = \frac{\Delta\lambda_{\text{limb}} - \Delta\lambda_{\text{centre}}}{f(\theta)_{\text{limb}} - f(\theta)_{\text{centre}}}$$

$\Delta\lambda^{(r)}_{e}$, being constant over the whole disk, is eliminated by taking differences and no reference to terrestial wavelengths is required.

The discussion of an extensive material of observations reveals that within the range of wavelengths from about $\lambda = 3\,700$ Å to $\lambda = 6\,600$ Å the ratio $\Delta\lambda_i/\lambda$ is practically constant. We believe that this result emphasises the fundamental significance of the interaction displacement.

There are various sets of observations available for investigating the wavelength dependency of $\Delta\lambda_i$:

1) An extensive set of observations by W. ADAMS [2] at Mt. Wilson, containing altogether measurements of 470 lines of various elements between $\lambda = 3\,742$ Å and $\lambda = 6\,573$ Å at the Sun's centre and at a clearly defined position near the limb.

2) Observations of FeI lines by St. JOHN [3] at Mt. Wilson made specially for the purpose of investigating the existence of the relativistic red shift.

3) Observations by EVERSHED and assistants of FeI, MnI, and TiI lines [4*a*, *b*] made principally at Kodaikanal and later at Ewhurst.

4) Observations at Potsdam by E. FINLAY-FREUNDLICH in collaboration with v. BRUNN and BRUCK [5], of relative shifts of a particularly sharp FeI multiplet around $\lambda = 4\,426$ Å in 72 positions over the whole solar disk.

For various reasons however, only W. ADAMS' observations give a substantial contribution to the problem under consideration. The measures of St. JOHN do not include many limb observations ; moreover, the " limb ", in this case, is not accurately defined, since the tabulated shifts are mean values derived from several individual plates, some of which are obtained from an image formed by the 60-foot tower telescope, and others using the 150-foot telescope. The lack of accurate information regarding the position of the slit relative to the limb is fatal with regard to the determination of $\Delta\lambda_i$, due to the rapid increase of the factor $f(\theta, R/l_0)$ near the limb.

The Kodaikanal and Ewhurst observations have been carefully analysed, but give very inconsistent results, of which the observers were aware but unable to explain. We are inclined to believe that this inconsistency is due to the small size of the solar image employed ; the resolution is not sufficient to yield an accurate determination of $f(\theta, R/l_0)$ at a point on the disk close to the limb.

The Potsdam observations, finally, yield a very accurate determination of $\Delta\lambda_i$; but only for a small wavelength region at $\bar{\lambda} = 4\,426$ Å around which the lines of the FeI multiplet crowd. These, together with the Oxford measures, will be considered later.

We shall first discuss the results obtained from a detailed analysis of the observations by W. Adams, made with the 60-foot tower telescope at Mt. Wilson. From the mean limb-centre displacements (Δ-values) of 470 spectral lines measured, the following 7 sets containing 306 lines in all, were chosen as being suited to the investigation of the wavelength dependency of $\Delta\lambda_i$:

127	FeI	lines	between	3 748 Å	and	6 569 Å
42	TiI	»	»	3 753 Å	»	5 953 Å
36	CaI	»	»	4 095 Å	»	6 573 Å
33	TiII	»	»	3 742 Å	»	5 337 Å
26	FeII	»	»	3 846 Å	»	6 516 Å
23	NiI	»	»	3 776 Å	»	6 191 Å
19	MnI	»	»	3 926 Å	»	6 022 Å

For all other elements, the numbers of observations are not numerous enough to warrant discussion.

For every element in the above list, the lines were combined into groups of increasing wavelength, and in every case they seemed to obey a linear relationship(*). Accordingly, the Δ-values were weighted by the number of individual measures made on each line, and a least-squares solution carried out on the conditional equation $\Delta = a.\lambda$, to determine the constant a for each element.

Then, using equation (2), we find :

$$\Delta\lambda_i = \frac{\Delta}{f(86^\circ 2', 10^2) - 1}$$

The value of $\theta = 86^\circ 2'$ is chosen in accordance with W. Adams' statement that his observations were made at 0.2 mm. inside the extreme limb of the solar image which had a diameter of 170 mm. The value $R/l_0 = 10^2$ is retained as the best value available for this ratio on the basis of the results obtained in our earlier paper [1].

* This fact was deduced by W. Adams himself in a less detailed analysis of his results.

The results of this analysis yielding $\Delta\lambda_i = c.\lambda$, with $c = \frac{a}{7.86}$ are illustrated in the Figs 1 (*a*) and 1 (*b*) ; the graphs reveal with surprising accuracy two important facts :

1) The observations for each element are well represented by a straight line passing through the zero point $\lambda = 0$. The observed values never differ from the

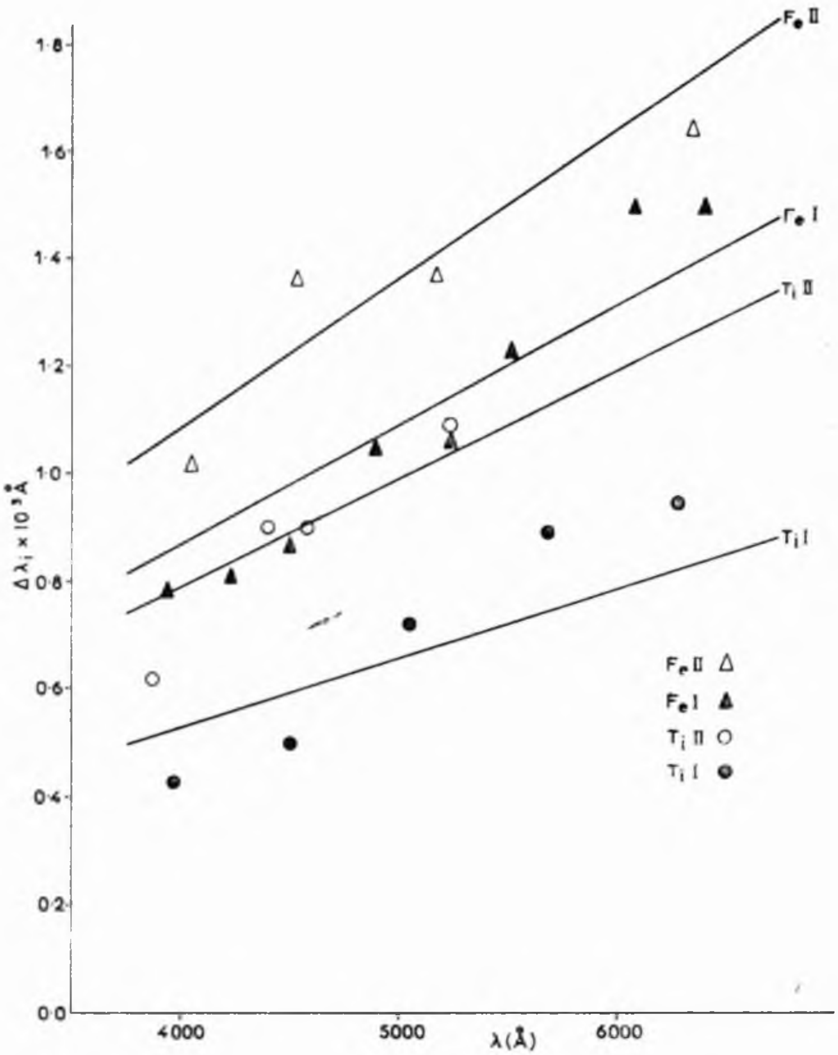

FIG. 1 (*a*).

Graphs of $\frac{\Delta\lambda_i}{\lambda} = c$ for different elements observed by W. ADAMS.

computed line by more than 0.2×10^{-3} Å. This apparently astonishing accuracy is not as surprising as it may seem ; for in our formula (2) the factor $f(86°2', 10^2) = 8.86$ enters as a natural weight which causes the scatter to be reduced to $\frac{1}{7.86} = 0.12$ of the observed amount — naturally, granting the working hypothesis to be true. A better representation of W. ADAMS' observations can be obtained, by admitting for instance an additional unknown constant or higher order terms in to the conditional equation. However, the observations of W. ADAMS, dating nearly 50 years back and not confined to a few selected good lines, may be affected

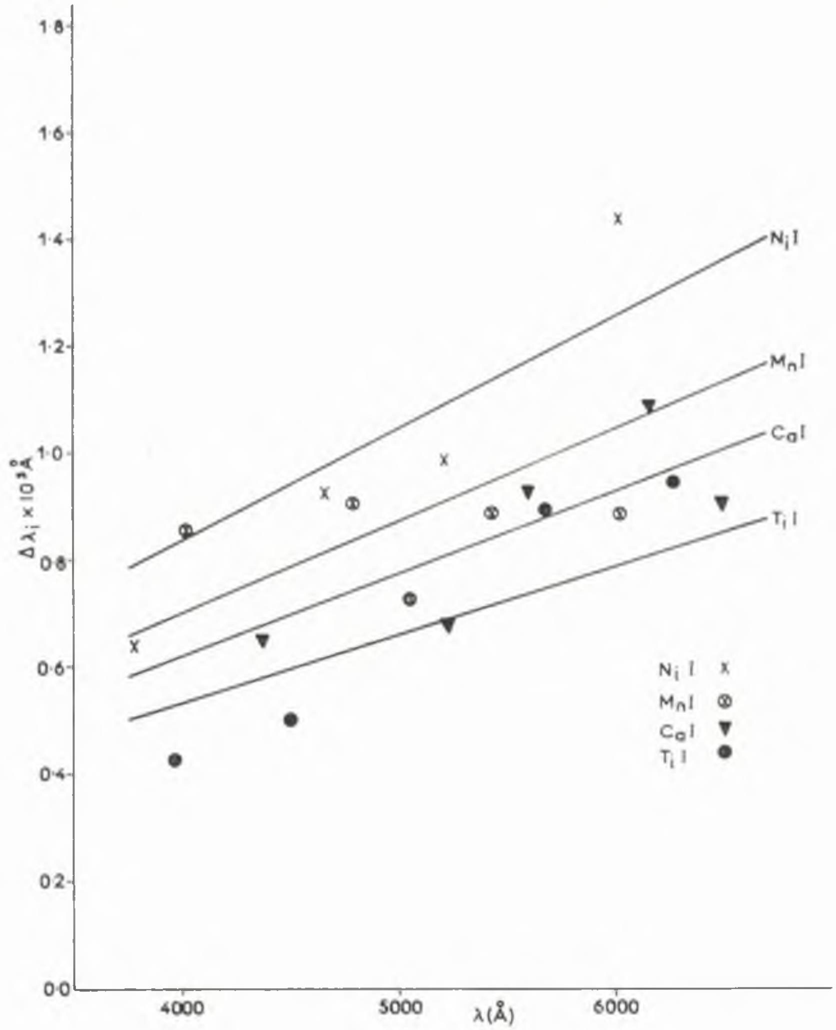

FIG. 1 (*b*).

Graphs of $\frac{\Delta\lambda}{\lambda} = c$ for different elements observed by W. ADAMS.

by small systematic and larger accidental errors than the latest Oxford observations. We can therefore not expect the terms $\Delta\lambda_c^{(r)}$ in equation (1) to cancel out completely when the expression (2) is formed, so that all straight lines will tend to pass accurately through the point $\lambda = 0$. In view of these uncertainties we feel there is no justification for employing other representations.

2) The amount of the interaction displacement $\Delta\lambda_i$ increases quite clearly with increasing ionization potential of the elements concerned. All values for the FeII lines lie above the values of FeI lines ; and similarly the TiII values are larger than

the corresponding values of TiI (*). Such an increase must become manifest if the lines of ionized atoms, as is to be expected, emerge from greater depths i. e. from layers of higher temperature in the Sun.

If we assume for the shifts a fourth-power temperature dependency, as suggested by one of the authors in a preceding paper [6], the increase in $\frac{\Delta\lambda_i}{\lambda}$ is found to be compatible with an increase of the *temperature* with increasing depth in the solar atmosphere as models of the sun's atmosphere lead us to expect. Using KING's results [7] (based on intensity measurements, made both in the solar spectrum and in an electric furnace) that the mean temperature at which TiI lines of low excitation potential originate in the solar " reversing-layer " is

$$T = 4\,400^{\circ}\,K \pm 100^{\circ},$$

and in addition using FREUNDLICH's red shift formula :

$$\Delta\lambda_i/\lambda = AT^4.l_0$$

(see [6]) with $A.l_0 = 3.52 \times 10^{-22}$ derived from the red shift of the TiI lines, the values given in Table I result :

TABLE I

ELEMENT	FeII	FeI	NiI	TiII	MnI	CaI	TiI
$\frac{\Delta\lambda_i}{\lambda} \times 10^7$ (obs.)	2.727	2.187	2.104	1.980	1.755	1.550	1.317
T°K. (calc.)	5 280	4 990	4 940	4 870	4 720	4 580	4 400

We wish to emphasise that the steady increase of the temperature, not the absolute values, is the essential feature of Table I. In view of this result, we are now inclined to believe that :

1) The tendency of the observed displacements $\Delta\lambda_{obs}$ to decrease initially slightly before finally increasing rapidly close to the limb, — which we referred to in a footnote in our 1st paper, — is due to the influence of the decrease in optical depth as we move outward along a solar radius ; thus initially the temperature dependency which enters with a high (most probably the 4th) power predominates. As the limb is approached, however, the rapid increase of the factor $f(\theta, R/l_0)$ when $\theta \rightarrow \frac{\pi}{2}$ determines the increase of the observed red shift. This is, however, a second order effect, and does not strongly affect the value chosen for R/l_0, predominant near the limb.

* Similar results are indicated foo VI and VII, but rely on only a few observations.

2) The scatter of the Y_r values (see [1]: Table V) by $\pm 5.10^{-4}$ Å which we thought to be accidental, is probably not so, but real and governed by the temperature at which each spectral line is actually produced.

An accurate check on the values in Table I is not possible, due to the uncer-

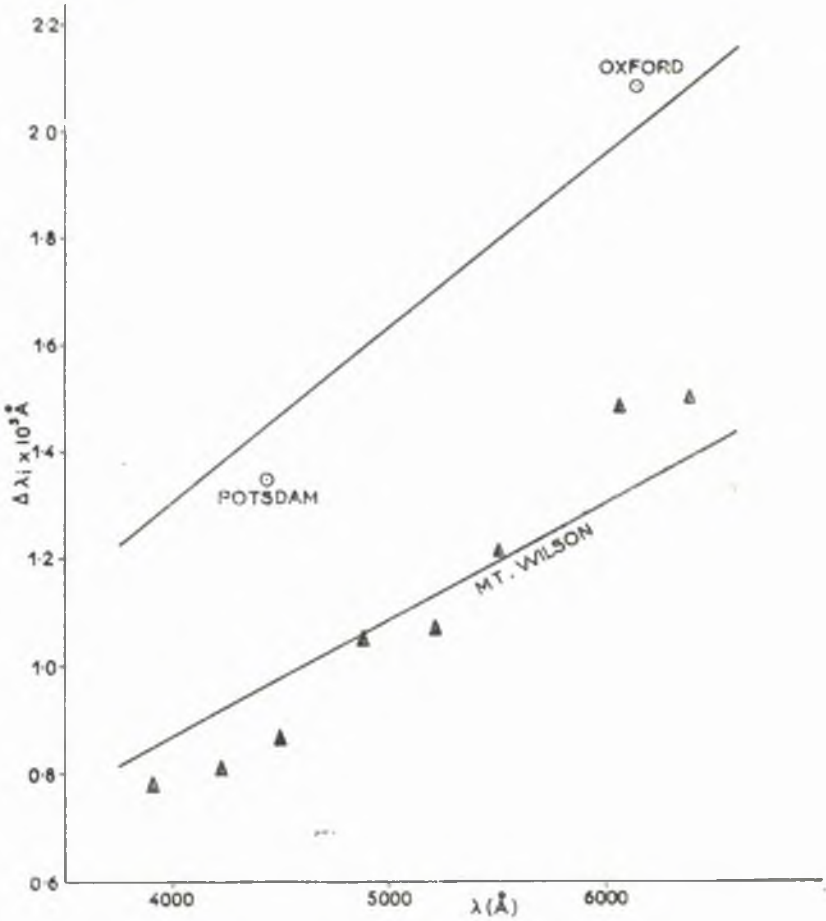

FIG. 2.
Comparaison of measures on FeI lines by different observers.

tainty in most of the physical parameters which are involved. But the measurement of relative limb-centre shifts should, when the nature of the interaction displacement is safely established, be able to provide a novel means of investigating the stratification of the various elements in the Sun's atmosphere. This effect is in character similar to the relative red shifts discovered by W. ADAMS and MACCORMACK in the spectra of giant stars, such as $\varkappa$ Orionis, between the spec-lines emerging from greater depths than the lines produced in the top level of their atmospheres [8]: this has been pointed out earlier [6] by one of the authors.

Now let us compare W. ADAMS' measures of the 127 FeI lines with the accurate values obtained at Potsdam and Oxford. Least-squares solutions of the latter sets yield the following results for the two groups of 9 FeI lines :

TABLE II

	$\bar{\lambda}$	$\Delta\lambda i$
Oxford	6 130 Å	2.09×10^{-3} Å
Potsdam	4 426 Å	1.35×10^{-3} Å

These observations are represented by the linear relationship $\Delta\lambda_i = c.\lambda$ in Fig. 2 which allows us to compare the slope of the lines thus obtained. The slopes agree very well, but the two straight lines do not coincide. The latter fact is probably due to the difficulty which exists on principle in comparing *quantatively* values derived from different sources. It is true, the two lines appear displaced relative to each other by not more than 0.5×10^{-3} Å over the wavelength region measured ; nevertheless, this difference is significant compared with the accuracy with which other observations fit the computed straight lines. It is easily shown, however, that a guiding error of only 1/2 mm. in the position of the spectrograph slit relative to the limb will completely account for this difference. The value of $f(\theta, R/l_0)$ is extremely sensitive to a small difference of this order very close to the limb : for on an image of 85 mm. radius as used by W. ADAMS, the difference corresponds to a change in θ of just over 3°, or of only 5″ of arc on the sun's disk.

A fact which might be mentioned in favour of this explanation for the difference between the two straight lines in Fig. 2. is that the measurements of a MnI triplet, which is contained in the Mt. Wilson and also in the Oxford observations, give self-consistent values for the limb-centre displacements in each set ; for the Mt. Wilson observations the value is $+ 7.10^{-3}$ Å, for Oxford $+ 8.10^{-3}$ Å. Since the latter value is derived from observations at the limb not nearer than $\theta \simeq 80°$, the slightly smaller value from W. ADAMS' material indicates that he must have actually observed on the average more than 0.2 mm. within the limb.

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ON THE RED SHIFT OF THE SOLAR LINES III

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Sommaire. — *Il est prouvé que la nouvelle interprétation du déplacement vers le rouge (suggérée d'abord par un des auteurs, à partir d'une étude des spectres stellaires,* 1954) *permet de rendre compte d'une façon complète, sans résidu, des observations : les seules mesures de déplacements absolus en longueur d'onde (soleil-arc) actuellement utilisables, c'est-à-dire les mesures interférométriques effectuées par Miss* Adam *à Oxford* (1948) *; et les différences relatives très complètes (bord-centre) observées par* W. Adams *au Mont Wilson* (1910). *L'existence de cet effet physique non établi ne laisse aucune place pour un déplacement vers le rouge constant, tel que celui prédit par la théorie générale de la relativité.*

Abstract. — *The new interpretation of the red shift first suggested by one of the authors from a study of stellar spectra* (1954) *is shown to account completely, without any remainder, for the only set of absolute (sun-arc) wavelength displacements presently available, i.e. interferometric measurements made by Miss* Adam *at Oxford* (1948) *— and for the extensive relative (limb-centre) differences observed by* W. Adams *at Mt. Wilson* (1910). *The existence of this unestablished physical effect leaves no room for the constant red shift predicted by the General Theory of Relativity.*

Резюме. — Доказано, что новое истолкование красного смещения (предложенное сначала одним из авторов на основании изучения звездных спектров в 1954 г.) позволяет без остатка представить наблюдения. Под последними надо понимать, во-первых, единственные годные в настоящее время измерения абсолютных смещений по длине волны (солнце-дуга), т.е. интерферометрические измерения, сделанные мисс Адам в Оксфорде (1948), а во-вторых, очень большое число относительных разностей (крайцентр), наблюденных В. Адамсом в обсерватории Маунт-Вилсон (1910). Существование этого неустановленного физического явления не оставляет места для постоянного красного смещения, предсказанного общей теорией относительности.

1. Introduction.

The hypothesis [4] that red shifts observed in certain types of stellar spectra and in the solar spectrum are compatible with a formula of the form

$$\frac{\Delta\nu}{\nu} = -\mathrm{A.T}^4\, l \qquad (1)$$

where A is a constant and l the length of path traversed by a photon of frequency ν through a radiation field of temperature T — has had further support from a more detailed re-analysis by the authors [5, 6] of existing measurements of solar wavelength displacements.

However, there still remains the particularly objectionable fact that an unaccountable constant term appears in the analysis of Miss Adam's data on the sun-arc shifts of selected wavelengths around 6100 Å [1], whose reality, though its value is only about 1/5 of the gravitational shift predicted by the General Theory of Relativity (see [4]), seems to be safely established. We have now realised from our study of solar spectrum wavelengths that this unexplained term is not an independent effect but actually constitutes a complementary part of the hypothetical effect itself ; it arises from the fact that the solar atmosphere is in a state of radiative equilibrium.

The following discussion shall demonstrate how this new result can be obtained directly from the working hypothesis (1).

2. Discussion.

We start from an equivalent form of equation (1) :

$$\frac{\Delta\lambda}{\lambda} = C.I_\lambda.l. \tag{2}$$

in which the sun-arc wavelength shift $\Delta\lambda$, the continuum intensity I_λ, and the length of path l all refer to a given disk position (specified by the angle θ) ; C is a new constant of proportionality analogous to the coefficient A in [4]. I_λ as well as T^4 are both quantities proportional to the *density of the radiation field* — the basic physical parameter postulated to describe the hypothetical effect.

Equation (2) may also be written as :

$$\frac{\Delta\lambda}{\lambda} = C\, I_\lambda(o)\, l_0 \frac{I_\lambda(\theta)}{I_\lambda(o)} \frac{l}{l_0} \tag{3}$$

where $I_\lambda(o)$ and l_0 both refer to the centre of the sun's disk.

According to the theory of radiative equilibrium the observed limb-darkening of the continuous radiation at the surface of the sun is well represented over all but the extreme edge of the disk by the formula :

$$\frac{I_\lambda(o, \theta)}{I_\lambda(o, o)} = 1 - u + u \cos\theta \tag{4}$$

u represents the limb-darkening coefficient whose value is near to 0.60. This coefficient is a function of wavelength, but its variations with wavelength (and with optical depth) are of secondary importance with regard to the present discussion. The blanketing-effect of the surface layer of the solar atmosphere can account for the slightly lower value ($u = 0.55$) actually observed [7]. We also know (see [5]) that to a high degree of approximation within the range $0 \leqslant \theta \leqslant 80^\circ$ covered by Miss Adam's observations :

$$\frac{l}{l_0} = f(\theta, R/l_0) \simeq \sec\theta \tag{5}$$

Substituting the relations (4) and (5) into (3) yields :

$$\Delta\lambda \simeq [C\lambda I_\lambda(o)\, l_0]\, u + [C.\lambda I_\lambda(o) l_0]\, (1 - u) \sec \theta. \tag{6}$$

Thus, provided that the working hypothesis is correct, the structure of the radiation field in radiative equilibrium leads necessarily to a red shift consisting of a *constant*(*) *term proportional to* u, and a *variable term* (*) *proportional to* $(1-u)$. It has already been shown in [4] that an equation of this special form gives an extremely accurate representation of Miss Adam's data but we have not yet verified the fact that the two terms yield a ratio equal to $\frac{u}{1-u}$.

The result obtained formerly (see [4] p. 313) from a least-squares solution of the Oxford observations for the mean red shift $\Delta\lambda$ expressed in milli-angstroms (mÅ), gave

$$\Delta\lambda = 2.72 + 1.85 \sec \theta. \tag{7}$$

By comparing (6) and (7) we obtain

$$\frac{u}{1-u} = \frac{2.72}{1.85} = 1.47,$$

from which follows the value $u = 0.60$.

A more accurate formula in which the curvature of the sun's outer layers is accounted for by employing the function $f(\theta, R/l_0)$ instead of sec θ (see [5]) yields for $R/l_0 = 100$ the equation

$$\Delta\lambda = 2.47 + 2.04\, f(\theta, 100).$$

In this case $\frac{u}{1-u}$ is $= 1.21$ and so $u = 0.55$, in full accordance with the limb-darkening observations. The constant term 2.72 resp. 2.47 which formerly could not be explained is now an organic part of the whole interpretation. It must, however, be noted that while every spectral line in the Oxford set of observations (when individually treated) gives effectively the *same value* for the constant $\Delta\lambda_i$ in the basic equation

$$\Delta\lambda = \Delta\lambda_c + \Delta\lambda_i\, f(\theta, R/l_0) \tag{7'}$$

by which the observed shifts have been represented (see [6]), the values of the individual $\Delta\lambda_c$ reveal an average scatter of nearly ± 3 mA around the mean value employed in the above discussion. This is to be expected, for the constant term is known to contain appreciable observational errors [2] in addition to all the uncertainties that affect the absolute wavelengths of solar lines. It may, however, be assumed that the mean value derived by averaging the observed centre-arc shifts

(*) i.e. with respect to the parameter θ.

of all 14 lines will not be seriously affected. To summarise : the observed red shifts of solar lines can be fully represented, *without remainder*, by the new hypothesis based on formula (2).

Let us secondly apply the same hypothesis to the limb-centre differences, observed by W. ADAMS [3] and see whether it is also compatible with these observations.

We obtain directly, by subtraction, from formula (2) :

$$\Delta = \Delta\lambda(\theta) - \Delta\lambda(o) = C\lambda I_\lambda(\theta)l - C\lambda I_\lambda(o)l_0$$

or

$$\Delta = \Delta\lambda(o)\left[\frac{I_\lambda(\theta)}{I_\lambda(o)}\cdot\frac{l}{l_0} - 1\right]. \tag{8}$$

In this case we can investigate the wavelength dependency of the ratio $\frac{I_\lambda(\theta)}{I_\lambda(o)}$, and establish whether (2) is valid for the whole of the visible range.

An analysis of W. ADAMS' data [3] has yielded the result that a highly accurate representation of the wavelength dependency of the limb-centre differences is given by $\Delta = a\lambda + b$... (9). The results are tabulated in Table I for 6 different elements for which measures of more than 20 lines had been made. With only one exception, which is not significant on account of the scarcity of the measures and their distribution in the visible range, the value for b results always negative ; this indicates its systematic character and consequently it must be regarded as real — a fact which was not acknowledged in our second paper in this series [6]. Comparing (8) and (9) yields the equations

$$a\lambda = \Delta\lambda(o)\frac{I_\lambda(\theta)}{I_\lambda(o)}\frac{l}{l_0}.$$

$$b = -\Delta\lambda(o)$$

from which follows by using (5) the equation :

$$\frac{I_\lambda(\theta)}{I_\lambda(o)} = -\frac{a}{bf(0,\, R/l_0)}\cdot\lambda. \tag{10}$$

In accordance with previous considerations described fully in [5] and [6] we choose $R/l_0 = 100$ and define the limb value for θ as corresponding to a position 0.7 mm inside the extreme limb of the sun. The coefficients $-\left(\frac{a}{bf(\theta, 100)}\right)$ can then be calculated ; they are tabulated in the last column of Table I. Table II gives the values of $\frac{I_\lambda(\theta)}{I_\lambda(o)}$ derived from (10) for 3 wavelengths covering the visible range from $\lambda = 4000$ to 6000 Å. Two sets of means of these figures — weighted according to the number of lines belonging to each element — are quoted, and in

TABLE I

Element	Total No. of Lines.	$\Delta = a\lambda + b$ $a \times 10^6$	b (mÅ)	$-\frac{1}{f(\theta, 100)} \cdot \frac{a}{b}$ $\times 10^3$/Å
—	—	—	—	—
Fe I	127	+ 2.58	— 4.5	+ 0.091
Ti I	42	2.16	— 5.4	0.064
Ca I	36	1.40	— 1.0	0.222
Ti II	33	2.64	— 4.9	0.086
Fe II	26	1.66	+ 2.5	— 0.105
Ni I	23	2.79	— 5.8	0.076

TABLE II

Element	Total No. of Lines.	λ 4 000	$\frac{I_\lambda)\theta)}{I_\lambda (o)}$ λ 5 000	λ 6 000
—	—	—	—	—
Fe I	127	0.36	0.46	0.55
Ti I	42	0.25	0.31	0.38
Ca I	36	0.88	1.10	1.32
Ti II	33	0.34	0.42	0.51
Fe II	26	— 0.42	— 0.52	— 0.63
Ni I	23	0.30	0.38	0.45
Weighted Mean.		0.33	0.42	0.50
do. (excluding Ca I and Fe II).		0.26	0.33	0.39
Observed Limb-darkening. $\frac{I_\lambda (\theta)}{I_\lambda (o)}$		0.26	0.38	0.46

the last row are added for comparison the extrapolated values of this ratio taken from Peyturaux' limb-darkening observations [8]. The agreement between these observed values and the values derived from the red shift observations on the basis of their new interpretation is seen to be very close indeed.

3. Conclusions.

Within the limited accuracy of available spectroscopic observations, the new hypothesis concerning the origin of the observed solar red shift suffices to account completely for the best available data on solar red shift measurements, quite indepedently of the results deduced from stellar observations ; in so doing it overcomes the difficulty arising from the presence of an hitherto unexplained constant term remaining in the discussion of Miss ADAM's data. Also W. ADAMS' observations of limb-centre differences yield results in full agreement with the new interpretation of the observed red shifts.

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